Real-World Application

Surface temperatures on Mars vary from $-128^\circ C$ during polar night to $27^\circ C$ at the equator during midday at the closest point in orbit to the sun. Find the difference between the highest value and the lowest value in this temperature range.

Source: Mars Institute

This problem appears as Exercise 73 in Section 2.3.
In this section, we extend the set of whole numbers to form the set of integers. You have probably already used negative numbers. For example, the outside temperature could drop to negative five degrees and a credit card statement could indicate activity of negative forty-eight dollars.

To create the set of integers, we begin with the set of whole numbers, 0, 1, 2, 3, and so on. For each number 1, 2, 3, and so on, we obtain a new number the same number of units to the left of zero on a number line.

For the number 1, there is the opposite number \(-1\) (negative 1).
For the number 2, there is the opposite number \(-2\) (negative 2).
For the number 3, there is the opposite number \(-3\) (negative 3), and so on.

The integers consist of the whole numbers and these new numbers. We illustrate them on a number line as follows.

The integers to the left of zero on the number line are called negative integers and those to the right of zero are called positive integers. Zero is neither positive nor negative and serves as its own opposite.

**Integers**

The integers: \(\ldots, -5, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5, \ldots\)

**Integers and the Real World**

Integers correspond to many real-world problems and situations. The following examples will help you get ready to translate problem situations to mathematical language.

**EXAMPLE 1**  Tell which integer corresponds to this situation: Researcher Robert Ballard discovered the wreck of the *Titanic* 12,500 ft below sea level.

*Source:* Office of Naval Research

12,500 ft below sea level corresponds to the integer \(-12,500\).
EXAMPLE 2  Tell which integers correspond to this situation: Elaine reversed the disc in her DVD player 17 min and then advanced it 25 min.

The integers −17 and 25 correspond to the situation. The integer −17 corresponds to the reversing and 25 corresponds to the advancing.

Do Exercises 1–5.

Order on the Number Line

Numbers are written in order on the number line, increasing as we move to the right. For any two numbers on the line, the one to the left is less than the one to the right.

Since the symbol < means “is less than,” the sentence −5 < 9 is read “−5 is less than 9.” The symbol > means “is greater than,” so the sentence −4 > −8 is read “−4 is greater than −8.”

EXAMPLES  Use either < or > for □ to form a true sentence.

3. −9 □ 2  Since −9 is to the left of 2, we have −9 < 2.
4. 7 □ −13  Since 7 is to the right of −13, we have 7 > −13.
5. −19 □ −6  Since −19 is to the left of −6, we have −19 < −6.

Do Exercises 6–9 on the next page.

Absolute Value

From the number line, we see that some integers, like 5 and −5, are the same distance from zero.

How far is 5 from 0? How far is −5 from 0? Since distance is never negative (it is “nonnegative,” that is, either positive or zero), it follows that both 5 and −5 are 5 units from 0.

The absolute value of a number is its distance from zero on a number line. We use the symbol |x| to represent the absolute value of a number x.

Like distance, the absolute value of a number is never negative; it is always either positive or zero.

Tell which integers correspond to each situation.

1. The halfback gained 8 yd on first down. The quarterback was sacked for a 5-yd loss on second down.

2. Temperature High and Low.
The highest recorded temperature in Nevada is 125°F (degrees Fahrenheit) on June 29, 1994, in Laughlin. The lowest recorded temperature in Nevada is 50°F below zero on January 8, 1937, in San Jacinto.

Sources: National Climatic Data Center, Asheville, NC, and Storm Phillips, STORMFAX, INC.

3. Stock Decrease.  The stock of Wendy’s decreased from $41 per share to $38 per share over a recent period.

Source: The New York Stock Exchange

4. At 10 sec (seconds) before liftoff, ignition occurs. At 148 sec after liftoff, the first stage is detached from the rocket.

5. Jacob owes $137 to the bookstore. Fortunately, he has $289 in a savings account.

Answers on page A-4
Find the absolute value of each number.
6. \(|-3|\) The distance of \(-3\) from 0 is 3, so \(|-3| = 3\).
7. \(|25|\) The distance of 25 from 0 is 25, so \(|25| = 25\).
8. \(|0|\) The distance of 0 from 0 is 0, so \(|0| = 0\).

To find a number’s absolute value:
1. If a number is positive or zero, use the number itself.
2. If a number is negative, make the number positive.


d Opposites

Recall that the set of integers can be represented on a number line. Given a number on one side of 0, we can get a number on the other side by reflecting the number across zero. For example, the reflection of 2 is \(-2\). We can read \(-2\) as “negative 2” or “the opposite of 2.”

The opposite of a number \(x\) is written \(-x\) (read “the opposite of \(x\”).

EXAMPLE 9 If \(x\) is \(-3\), find \(-x\).

To find the opposite of \(x\) when \(x\) is \(-3\), we reflect \(-3\) to the other side of 0.

When \(x = -3\), \(-x = -(\-3)\). We substitute \(-3\) for \(x\). We have \(-(\-3) = 3\). The opposite of \(-3\) is 3.

EXAMPLE 10 Find \(-x\) when \(x\) is 0.

When we try to reflect 0 “to the other side of 0,” we go nowhere:

\(\ -x = 0 \) when \( x = 0 \). The opposite of 0 is 0.

In Examples 9 and 10, the variable was replaced with a number. When this occurs, we say that we are evaluating the expression.

EXAMPLE 11 Evaluate \(-x\) when \(x\) is 4.

To find the opposite of \(x\) when \(x\) is 4, we reflect 4 to the other side of 0.

We have \(-(4) = -4\). The opposite of 4 is \(-4\).
Do Exercises 14–16.

A negative number is sometimes said to have a negative sign. A positive number is said to have a positive sign, even though it rarely is written in.

**EXAMPLES** Determine the sign of each number.

12. \(-7\)  Negative  
13. \(23\)  Positive

Replacing a number with its opposite, or additive inverse, is sometimes called changing the sign.

**EXAMPLES** Change the sign (find the opposite, or additive inverse) of each number.

14. \(-6\) \(-(-6) = 6\)  
15. \(-10\) \(-(-10) = 10\)  
16. \(0\) \(-0 = 0\)  
17. \(-14\) \(-(-14) = -14\)

**Do Exercises 17–20.**

**EXAMPLE 18** If \(x\) is 2, find \(-(-x)\).

We replace \(x\) with 2:

\[\begin{align*}
-(-x) & \quad \text{Read “the opposite of the opposite of } x” \\
= -(-2) & \quad \text{We copy the expression, replacing } x \text{ with 2}
\end{align*}\]

The opposite of the opposite of 2 is 2, or \(-(-2) = 2\).

**EXAMPLE 19** Evaluate \(-(-x)\) for \(x = -4\).

We replace \(x\) with \(-4\):

\[\begin{align*}
-(-x) & \quad \\
= -(-(-4)) & \quad \text{Using an extra set of parentheses to avoid notation like } -(-4) \\
= -(-4) & \quad \text{Changing the sign of } -4 \\
= -4 & \quad \text{Changing the sign of } 4
\end{align*}\]

Thus, \(-(-(-4)) = -4\).

When we change a number’s sign twice, we return to the original number.

**Do Exercises 21–24.**

It is important not to confuse parentheses with absolute-value symbols.

**EXAMPLE 20** Evaluate \(-|x|\) for \(x = 2\).

We replace \(x\) with 2:

\[\begin{align*}
-|x| & \quad \\
= -|2| & \quad \text{Replacing } x \text{ with 2} \\
= -2 & \quad \text{The absolute value of } -2 \text{ is } 2
\end{align*}\]

Thus, \(-|2| = -2\).

Note that \(-(-2) = 2\), whereas \(-|-2| = -2\).

**Do Exercises 25 and 26.**

In each case draw a number line, if necessary.

14. Find \(-x\) when \(x = 1\).

15. Find \(-x\) when \(x = -2\).

16. Evaluate \(-x\) when \(x = 0\).

Change the sign. (Find the opposite, or additive inverse.)

17. \(-4\)  
18. \(-13\)

19. \(39\)  
20. \(0\)

21. If \(x\) is 7, find \(-(-x)\).

22. If \(x\) is 1, find \(-(-x)\).

23. Evaluate \(-(-x)\) for \(x = -6\).

24. Evaluate \(-(-x)\) for \(x = -2\).

25. Find \(-|-7|\).

26. Find \(-|-39|\).

*Answers on page A-4*
Tell which integers correspond to each situation.

1. **Pollution Fine.** In 2003, The Colonial Pipeline Company was fined a record $34 million for pollution.
   
   **Source:** Green Consumer Guide.com

2. **Highest and Lowest Temperatures.** The highest temperature ever created on earth was 950,000,000°F. The lowest temperature ever created was approximately 460°F below zero.
   
   **Source:** The Guinness Book of Records, 2004

3. The recycling program for Colchester once received $40 for a ton of office paper. More recently, they’ve had to pay $15 to get rid of a ton of office paper.

4. The space shuttle stood ready, 3 sec before liftoff. Solid fuel rockets were released 128 sec after liftoff.

5. At tax time, Janine received an $820 refund while David owed $541.

6. **Oceanography.** At a depth of 2438 meters researchers found the first hydrothermal vent ever seen by humans. This depth is approximately 8000 ft below sea level.
   
   **Source:** Office of Naval Research

7. **Geography.** Death Valley, California, is 280 ft below sea level. Mt. Whitney, the highest point in California, has an elevation of 14,491 ft.

8. **Geography.** The Dead Sea, between Jordan and Israel, is 1286 ft below sea level; Mt. Rainier in Washington State is 14,410 ft above sea level.

---

Use either < or > for □ to form a true sentence.

9. –8 □ 0
10. 7 □ 0
11. 9 □ 0
12. –7 □ 0
13. 8 □ –8
14. 6 □ –6
15. –6 □ –4
16. –1 □ –7
17. –8 □ –5
18. –5 □ –3
19. –13 □ –9
20. –5 □ –11
21. –3 □ –4
22. –6 □ –5
Find the absolute value.

23. |57| 24. |11| 25. |0| 26. |-4| 27. |-24|

28. |-36| 29. |53| 30. |54| 31. |-8| 32. |-79|

Find \(-x\) when \(x\) is each of the following.

33. \(-7\) 34. \(-6\) 35. 7 36. 6 37. 0

38. \(-1\) 39. \(-19\) 40. 50 41. 42 42. \(-73\)

Change the sign. (Find the opposite, or additive inverse.)

43. \(-8\) 44. \(-7\) 45. 7 46. 10 47. \(-29\)

48. \(-14\) 49. \(-22\) 50. 0 51. 1 52. \(-53\)

Evaluate \(-(-x)\) when \(x\) is each of the following.

53. 7 54. \(-8\) 55. \(-9\) 56. 3 57. \(-17\) 58. \(-19\)

59. 23 60. 0 61. \(-1\) 62. 73 63. 85 64. \(-37\)

Evaluate \(-|-x|\) when \(x\) is each of the following.

65. 47 66. 92 67. 345 68. 729

69. 0 70. 1 71. \(-8\) 72. \(-3\)
73. D_W Does \(-x\) always represent a negative number? Why or why not?

74. D_W Explain in your own words why \(-(-x) = x\).

**SKILL MAINTENANCE**

75. Add: 327 + 498. [1.2b]

76. Evaluate: 5^3. [1.9b]

77. Multiply: 209 \cdot 34. [1.5a]

78. Solve: 300 \cdot x = 1200. [1.7b]

79. Evaluate: 9^2. [1.9b]

80. Multiply: 31 \cdot 50. [1.5a]

81. Simplify: 5(8 - 6). [1.9c]

82. Simplify: 7(9 - 3). [1.9c]

**SYNTHESIS**

83. D_W If \(a > b\) is true, does it follow that \(-b > -a\) is also true? Why or why not?

84. D_W Does \(|x|\) always represent a positive number? Why or why not?

85. \(\square\) On your calculator list the sequence of keystrokes needed to find the opposite of the sum of 549 and 387.

86. \(\square\) On your calculator list the sequence of keystrokes needed to find the opposite of the product of 438 and 97.

Use either <, >, or = for \(\square\) to write a true sentence.

87. \(|-5| \square |-2|

88. |4| \square |-7|

89. \(|-8| \square |8|

Simplify.

90. \(-|3|\)

91. \(-|-8|\)

92. \(-|-2|\)

93. \(-|7|\)

Solve. Consider only integer replacements.

94. \(|x| = 7\)

95. \(|x| < 2\)

96. Simplify \(-(-x), -(\neg(-x)), \) and \(-(-(-(-x)))\).

97. List these integers in order from least to greatest.

\(2^{10}, -5, |-6|, 4, |3|, -100, 0, 2^7, 7^2, 10^2\)
2.2 ADDITION OF INTEGERS

a Addition

To explain addition of integers, we can use the number line. Once our understanding is developed, we will streamline our approach.

**ADDING INTEGERS**

To perform the addition \( a + b \), we start at \( a \), and then move according to \( b \).

a) If \( b \) is positive, we move to the right.

b) If \( b \) is negative, we move to the left.

c) If \( b \) is 0, we stay at \( a \).

**EXAMPLE 1** Add: \( 2 + (-5) \).

Start at 2.

Move 5 units to the left.

\( 2 + (-5) = -3 \)

**EXAMPLE 2** Add: \( -1 + (-3) \).

Start at -1.

Move 3 units to the left.

\( -1 + (-3) = -4 \)

**EXAMPLE 3** Add: \( -4 + 9 \).

Start at -4.

Move 9 units to the right.

\( -4 + 9 = 5 \)

Do Exercises 1–7.

You may have noticed a pattern in Example 2 and Margin Exercises 2 and 6. When two negative integers are added, the result is negative.

**ADDING NEGATIVE INTEGERS**

To add two negative integers, add their absolute values and change the sign (making the answer negative).

Add, using a number line.

1. \( 3 + (-4) \)

2. \( -3 + (-5) \)

3. \( -3 + 7 \)

4. \( -5 + 5 \)

For each illustration, write a corresponding addition sentence.

5.

\( -5 \)

\( -4 \)

\( -3 \)

\( -2 \)

\( -1 \)

\( 0 \)

\( 1 \)

\( 2 \)

\( 3 \)

\( 4 \)

6.

7.

\( -4 \)

\( -3 \)

\( -2 \)

\( -1 \)

\( 0 \)

\( 1 \)

\( 2 \)

\( 3 \)

\( 4 \)

\( 5 \)

Answers on page A-4
Add. Do not use a number line except as a check.

8. \(-5 + (-6)\)

9. \(-9 + (-3)\)

10. \(-20 + (-14)\)

11. \(-11 + (-11)\)

12. \(0 + (-17)\)

13. \(49 + 0\)

14. \(-56 + 0\)

Add, using a number line only as a check.

15. \(-4 + 6\)

16. \(-7 + 3\)

17. \(5 + (-7)\)

18. \(10 + (-7)\)

Answers on page A-4

**EXAMPLES** Add.

4. \(-5 + (-7) = -12\)
   \[\text{Think: Add the absolute values: } 5 + 7 = 12.\]
   Make the answer negative, \(-12\).

5. \(-8 + (-2) = -10\)
   \[\text{We can visualize the number line without actually drawing it.}\]

**Do Exercises 8–11.**

Note that the sum of two positive integers is positive, and the sum of two negative integers is negative.

When the number 0 is added to any number, that number remains unchanged. For this reason, the number 0 is referred to as the additive identity.

**EXAMPLES** Add.

6. \(-4 + 0 = -4\)

7. \(0 + (-9) = -9\)

8. \(17 + 0 = 17\)

**Do Exercises 12–14.**

When we add a positive integer and a negative integer, as in Examples 1 and 3, the sign of the number with the greater absolute value is the sign of the answer.

**ADDING POSITIVE AND NEGATIVE INTEGERS**

To add a positive integer and a negative integer, find the difference of their absolute values.

a) If the negative integer has the greater absolute value, the answer is negative.

b) If the positive integer has the greater absolute value, the answer is positive.

**EXAMPLES** Add.

9. \(3 + (-5) = -2\)
   \[\text{Think: The absolute values are 3 and 5. The difference is 2. Since the negative number has the larger absolute value, the answer is negative, } -2.\]

10. \(11 + (-8) = 3\)
   \[\text{Think: The absolute values are 11 and 8. The difference is 3. The positive number has the larger absolute value, so the answer is positive, } 3.\]

11. \(1 + (-6) = -5\)

12. \(-7 + 4 = -3\)

13. \(7 + (-3) = 4\)

14. \(-6 + 10 = 4\)

**Do Exercises 15–18.**

Sometimes \(-a\) is referred to as the additive inverse of \(a\). This terminology is used because adding any number to its additive inverse always results in the additive identity, 0.

\[-8 + 8 = 0, \quad 14 + (-14) = 0, \quad \text{and} \quad 0 + 0 = 0.\]
**ADDITION OF INTEGERS**

For any integer \( a \),

\[ a + (-a) = -a + a = 0. \]

(The sum of any number and its additive inverse, or opposite, is 0.)

**Do Exercises 19–22.**

Suppose we wish to add several numbers, positive and negative:

\[ 15 + (-2) + 7 + 14 + (-5) + (-12). \]

Because of the commutative and associative laws for addition, we can group the positive numbers together and the negative numbers together and add them separately. Then we add the two results.

**EXAMPLE 15** Add: \( 15 + (-2) + 7 + 14 + (-5) + (-12) \).

First add the positive numbers: \( 15 + 7 + 14 = 36 \).

Then add the negative numbers: \( -2 + (-5) + (-12) = -19 \).

Finally, add the results: \( 36 + (-19) = 17 \).

We can also add in any other order we wish, say, from left to right:

\[
15 + (-2) + 7 + 14 + (-5) + (-12) = 13 + 7 + 14 + (-5) + (-12) = 20 + 14 + (-5) + (-12) = 34 + (-5) + (-12) = 29 + (-12) = 17.
\]

**Do Exercises 23–25.**

**Study Tips**

**HELP SESSIONS**

- **Make the most of tutoring sessions by doing what you can ahead of time and knowing the topics with which you need help.**

- **Work on the topics before you go to the help or tutoring session.** Do not regard yourself as an empty cup that the tutor will fill with knowledge. The primary source of your ability to learn is within you. When students go to help or tutoring sessions unprepared, they waste time and, in many cases, money. Go to class, study the textbook, work exercises, and mark trouble spots. **Then** use the help and tutoring sessions to work on the trouble spots.

- **Do not be afraid to ask questions in these sessions!** The more you talk to your tutor, the more the tutor can help you.

- **Try being a “tutor” yourself.** Explaining a topic to someone else—a classmate, your instructor—is often the best way to master it.

**Add, using a number line only as a check.**

19. \( 5 + (-5) \)

20. \( -6 + 6 \)

21. \( -10 + 10 \)

22. \( 89 + (-89) \)

23. \( (-15) + (-37) + 25 + 42 + (-59) + (-14) \)

24. \( 42 + (-81) + (-28) + 24 + 18 + (-31) \)

25. \( -35 + 17 + 14 + (-27) + 31 + (-12) \)

**Answers on page A-4**
Add, using a number line.

1. \(-7 + 2\)  
2. \(1 + (-5)\)  
3. \(-9 + 5\)  
4. \(8 + (-3)\)  
5. \(-3 + 9\)

6. \(9 + (-9)\)  
7. \(-7 + 7\)  
8. \(-8 + (-5)\)  
9. \(-3 + (-1)\)  
10. \(-2 + (-9)\)

11. \(4 + (-9)\)  
12. \(-4 + 13\)  
13. \(-7 + 12\)  
14. \(-3 + 2\)

Add. Use a number line only as a check.

15. \(-3 + (-9)\)  
16. \(-3 + (-7)\)  
17. \(-6 + (-5)\)  
18. \(-10 + (-14)\)

19. \(5 + (-5)\)  
20. \(10 + (-10)\)  
21. \(-2 + 2\)  
22. \(-3 + 3\)

23. \(0 + 6\)  
24. \(7 + 0\)  
25. \(13 + (-13)\)  
26. \(-17 + 17\)

27. \(-25 + 0\)  
28. \(-43 + 0\)  
29. \(0 + (-27)\)  
30. \(0 + (-19)\)

31. \(-31 + 31\)  
32. \(12 + (-12)\)  
33. \(-8 + 0\)  
34. \(-11 + 0\)

35. \(9 + (-4)\)  
36. \(-7 + 8\)  
37. \(-4 + (-5)\)  
38. \(0 + (-3)\)

39. \(0 + (-5)\)  
40. \(10 + (-12)\)  
41. \(14 + (-5)\)  
42. \(-3 + 14\)
<p>| | |</p>
<table>
<thead>
<tr>
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<tbody>
<tr>
<td>43.</td>
<td>$-11 + 8$</td>
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<td>44.</td>
<td>$0 + (-34)$</td>
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<tr>
<td>45.</td>
<td>$-19 + 19$</td>
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<tr>
<td>46.</td>
<td>$-10 + 3$</td>
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<td>47.</td>
<td>$-16 + 6$</td>
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<td>48.</td>
<td>$-15 + 5$</td>
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<td>$-17 + (-7)$</td>
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<td>50.</td>
<td>$-15 + (-5)$</td>
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<td>51.</td>
<td>$11 + (-16)$</td>
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<td>52.</td>
<td>$-8 + 14$</td>
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<td>53.</td>
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<td>56.</td>
<td>$-14 + (-19)$</td>
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<td>57.</td>
<td>$-11 + 17$</td>
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<td>58.</td>
<td>$19 + (-19)$</td>
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<tr>
<td>59.</td>
<td>$-15 + (-7) + 1$</td>
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<tr>
<td>60.</td>
<td>$23 + (-5) + 4$</td>
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<tr>
<td>61.</td>
<td>$30 + (-10) + 5$</td>
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<tr>
<td>62.</td>
<td>$40 + (-8) + 5$</td>
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<td>63.</td>
<td>$-23 + (-9) + 15$</td>
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<tr>
<td>64.</td>
<td>$-25 + 25 + (-9)$</td>
</tr>
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<td>65.</td>
<td>$40 + (-40) + 6$</td>
</tr>
<tr>
<td>66.</td>
<td>$63 + (-18) + 12$</td>
</tr>
<tr>
<td>67.</td>
<td>$12 + (-65) + (-12)$</td>
</tr>
<tr>
<td>68.</td>
<td>$-35 + (-63) + 35$</td>
</tr>
<tr>
<td>69.</td>
<td>$-24 + (-37) + (-19) + (-45) + (-35)$</td>
</tr>
<tr>
<td>70.</td>
<td>$75 + (-14) + (-17) + (-5)$</td>
</tr>
<tr>
<td>71.</td>
<td>$28 + (-44) + 17 + 31 + (-94)$</td>
</tr>
<tr>
<td>72.</td>
<td>$27 + (-54) + (-32) + 65 + 46$</td>
</tr>
<tr>
<td>73.</td>
<td>$-19 + 73 + (-23) + 19 + (-73)$</td>
</tr>
<tr>
<td>74.</td>
<td>$35 + (-51) + 29 + 51 + (-35)$</td>
</tr>
</tbody>
</table>
75. **Dw** Explain in your own words why the sum of two negative numbers is always negative.

76. **Dw** A student states “−45 is bigger than −21.” What mistake do you think the student is making?

### Skill Maintenance

Subtract. [1.3d]

77. \(543 - 219\) 78. \(6314 - 2689\) 79. \(2891 - 1407\) 80. \(19876 - 14321\)

81. Write in expanded notation: 39,417. [1.1b]

82. Round to the nearest hundred: 746. [1.4a]

83. Round to the nearest thousand: 32,831. [1.4a]

84. Multiply: \(42 \cdot 56\). [1.5a]

85. Divide: \(288 \div 9\). [1.6c]

86. Round to the nearest ten: 3496. [1.4a]

### Synthesis

87. **Dw** Without using the words “absolute value,” explain how to find the sum of a positive number and a negative number.

88. **Dw** Why is it important to understand the associative and commutative laws when adding more than two integers at a time?

Add.

89. \(-[27] + (-[-13])\)

90. \([-32] + (-[15])\)

91. \(-3496 + (-2987)\)

92. \(497 + (-3028)\)

93. \(-7846 + 5978\)

94. \(-7623 + 4839\)

95. For what numbers \(x\) is \(-x\) positive?

96. For what numbers \(x\) is \(-x\) negative?

Tell whether each sum is positive, negative, or zero.

97. If \(n\) is positive and \(m\) is negative, then \(-n + m\) is _______.

98. If \(n = m\) and \(n\) is negative, then \(-n + (-m)\) is _______.

99. If \(n\) is negative and \(m\) is less than \(n\), then \(n + m\) is _______.

100. If \(n\) is positive and \(m\) is greater than \(n\), then \(n + m\) is _______.
2.3

SUBTRACTION OF INTEGERS

a  Subtraction

We now consider subtraction of integers. To find the difference $a - b$, we look for a number to add to $b$ that gives us $a$.

THE DIFFERENCE

The difference $a - b$ is the number that when added to $b$ gives $a$.

For example, $45 - 17 = 28$ because $28 + 17 = 45$. Let’s consider an example in which the answer is a negative number.

EXAMPLE 1  Subtract: $5 - 8$.

Think: $5 - 8$ is the number that when added to 8 gives 5. What number can we add to 8 to get 5? The number must be negative. The number is $-3$:

$$5 - 8 = -3.$$ That is, $5 - 8 = -3$ because $8 + (-3) = 5$.

Do Exercises 1–3.

The definition of $a - b$ above does not always provide the most efficient way to subtract. To understand a faster way to subtract, consider finding $5 - 8$ using a number line. We start at 5. Then we move 8 units to the left to do the subtracting. Note that this is the same as adding the opposite of 8, or $-8$, to 5.

Move 8 units to the left. Start at 5.

$5 - 8 = -3$

Look for a pattern in the following table.

<table>
<thead>
<tr>
<th>SUBTRACTIONS</th>
<th>ADDING AN OPPOSITE</th>
</tr>
</thead>
<tbody>
<tr>
<td>$5 - 8 = -3$</td>
<td>$5 + (-8) = -3$</td>
</tr>
<tr>
<td>$-6 - 4 = -10$</td>
<td>$-6 + (-4) = -10$</td>
</tr>
<tr>
<td>$-7 - (-10) = 3$</td>
<td>$-7 + 10 = 3$</td>
</tr>
<tr>
<td>$-7 - (-2) = -5$</td>
<td>$-7 + 2 = -5$</td>
</tr>
</tbody>
</table>

Do Exercises 4–7.

Perhaps you have noticed that we can subtract by adding the opposite of the number being subtracted. This can always be done.

Answers on page A-4
Equate each subtraction with a corresponding addition. Then write the equation in words.

8. \(3 - 10\)

9. \(9 - 5\)

10. \(-12 - (-9)\)

11. \(-12 - 10\)

12. \(-14 - (-14)\)

\[\text{Subtract.}\]

13. \(7 - 11\)

**EXAMPLES**  Equate each subtraction with a corresponding addition. Then write the equation in words.

2. \(-12 - 30;\) \[\begin{align*}
-12 - 30 &= -12 + (-30) & \text{Adding the opposite of 30}
\end{align*}\]

Negative twelve minus thirty is negative twelve plus negative thirty.

3. \(-20 - (-17);\) \[\begin{align*}
-20 - (-17) &= -20 + 17 & \text{Adding the opposite of -17}
\end{align*}\]

Negative twenty minus negative seventeen is negative twenty plus seventeen.

Do Exercises 8–12.

Once the subtraction has been rewritten as addition, we add as in Section 2.2.

**EXAMPLES**  Subtract.

4. \(2 - 6 = 2 + (-6)\) \[\begin{align*}
2 - 6 &= 2 + (-6) \\
&= -4
\end{align*}\]

The opposite of 6 is -6. We change the subtraction to addition and add the opposite.

Instead of subtracting 6, we add -6.

5. \(4 - (-9) = 4 + 9\) \[\begin{align*}
4 - (-9) &= 4 + 9 \\
&= 13
\end{align*}\]

The opposite of -9 is 9. We change the subtraction to addition and add the opposite.

Instead of subtracting -9, we add 9.

6. \(-4 - 8 = -4 + (-8)\) \[\begin{align*}
-4 - 8 &= -4 + (-8) \\
&= -12
\end{align*}\]

We change the subtraction to addition and add the opposite.

Instead of subtracting 8, we add -8.

7. \(10 - 7 = 10 + (-7)\) \[\begin{align*}
10 - 7 &= 10 + (-7) \\
&= 3
\end{align*}\]

We change the subtraction to addition and add the opposite.

Instead of subtracting 7, we add -7.

8. \(-4 - (-9) = -4 + 9\) \[\begin{align*}
-4 - (-9) &= -4 + 9 \\
&= 5
\end{align*}\]

Instead of subtracting -9, we add 9.

To check, note that \(5 + (-9) = -4\).

9. \(-7 - (-3) = -7 + 3\) \[\begin{align*}
-7 - (-3) &= -7 + 3 \\
&= -4
\end{align*}\]

Instead of subtracting -3, we add 3.

Check: \(-4 + (-3) = -7\).

Do Exercises 13–18.
When several additions and subtractions occur together, we can make them all additions. The commutative law for addition can then be used.

**EXAMPLE 10** Simplify: \(-3 - (-5) - 9 + 4 - (-6)\).

\[
-3 - (-5) - 9 + 4 - (-6) = -3 + 5 + (-9) + 4 + 6 \quad \text{Adding opposites}
\]

\[
= -3 + (-9) + 5 + 4 + 6 \quad \text{Using a commutative law}
\]

\[
= -12 + 15 \quad \frac{3}{}
\]

Do Exercises 19 and 20.

### Applications and Problem Solving

We need addition and subtraction of integers to solve a variety of applied problems.

**EXAMPLE 11** *Toll Roads.* The E-Z Pass program allows drivers in the Northeast to travel certain toll roads without having to stop to pay. Instead, a transponder attached to the vehicle is scanned as the vehicle rolls through a toll booth. Recently the Ramones began a trip to New York City with a balance of $12 in their E-Z Pass account. Their trip accumulated $15 in tolls, and because they overspent their balance, the Ramones had to pay $80 in fines and administrative fees. By how much were the Ramones in debt as a result of their travel on the toll roads?

*Source: State of New Jersey*

We solve by first subtracting the cost of the tolls from the original balance in the account. Then we subtract the cost of the fees and fines from the new balance in the account:

\[
12 - 15 = 12 + (-15) \quad \text{Adding the opposite of 15}
\]

\[
= -3,
\]

and

\[
-3 - 80 = -3 + (-80) \quad \text{Adding the opposite of 80}
\]

\[
= -83.
\]

The Ramones were $83 in debt as a result of their travel on toll roads.

Do Exercises 21 and 22.

### Study Tips

**USING THE ANSWER SECTION**

It is easy to become overdependent on the answer section. When using the answers listed in the back of this book, try not to “work backward” from the answer. If you find that you frequently require two or more attempts to answer an exercise correctly, you probably need to work more carefully and/or reread the section preceding the exercise set. Remember that on quizzes and tests you have only one attempt to answer each question.

Answers on page A-4
### Subtract.

<p>| | | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1. $2 - 7$</td>
<td>2. $3 - 8$</td>
<td>3. $0 - 8$</td>
<td>4. $0 - 9$</td>
</tr>
<tr>
<td></td>
<td>5. $-7 - (-4)$</td>
<td>6. $-6 - (-8)$</td>
<td>7. $-11 - (-11)$</td>
<td>8. $-6 - (-6)$</td>
</tr>
<tr>
<td></td>
<td>9. $13 - 17$</td>
<td>10. $14 - 19$</td>
<td>11. $20 - 27$</td>
<td>12. $30 - 4$</td>
</tr>
<tr>
<td></td>
<td>13. $-9 - (-4)$</td>
<td>14. $-7 - (-9)$</td>
<td>15. $-40 - (-40)$</td>
<td>16. $-9 - (-9)$</td>
</tr>
<tr>
<td></td>
<td>17. $7 - 7$</td>
<td>18. $9 - 9$</td>
<td>19. $7 - (-7)$</td>
<td>20. $4 - (-4)$</td>
</tr>
<tr>
<td></td>
<td>21. $8 - (-3)$</td>
<td>22. $-7 - 4$</td>
<td>23. $-6 - 8$</td>
<td>24. $6 - (-10)$</td>
</tr>
<tr>
<td></td>
<td>25. $-3 - (-9)$</td>
<td>26. $-14 - 2$</td>
<td>27. $1 - 9$</td>
<td>28. $2 - 8$</td>
</tr>
<tr>
<td></td>
<td>29. $-6 - (-5)$</td>
<td>30. $-4 - (-3)$</td>
<td>31. $8 - (-10)$</td>
<td>32. $5 - (-6)$</td>
</tr>
<tr>
<td></td>
<td>33. $0 - 10$</td>
<td>34. $0 - 23$</td>
<td>35. $-5 - (-2)$</td>
<td>36. $-3 - (-1)$</td>
</tr>
<tr>
<td></td>
<td>37. $-7 - 14$</td>
<td>38. $-9 - 16$</td>
<td>39. $0 - (-5)$</td>
<td>40. $0 - (-1)$</td>
</tr>
<tr>
<td></td>
<td>41. $-8 - 0$</td>
<td>42. $-9 - 0$</td>
<td>43. $7 - (-5)$</td>
<td>44. $7 - (-4)$</td>
</tr>
</tbody>
</table>
Exercise Set 2.3

45. $6 - 25$
46. $18 - 63$
47. $-42 - 26$
48. $-18 - 63$

49. $-72 - 9$
50. $-49 - 3$
51. $24 - (-92)$
52. $48 - (-73)$

53. $-50 - (-50)$
54. $-70 - (-70)$
55. $-30 - (-85)$
56. $-25 - (-15)$

Simplify.
57. $7 - (-5) + 4 - (-3)$
58. $-5 - (-8) + 3 - (-7)$
59. $-31 + (-28) - (-14) - 17$

60. $-43 - (-19) - (-21) + 25$
61. $-34 - 28 + (-33) - 44$
62. $39 + (-88) - 29 - (-83)$

63. $-93 - (-84) - 41 - (-56)$
64. $84 + (-99) + 44 - (-18) - 43$
65. $-5 - (-30) + 30 + 40 - (-12)$

66. $14 - (-50) + 20 - (-32)$
67. $132 - (-21) + 45 - (-21)$
68. $81 - (-20) - 14 - (-50) + 53$

Solve.

69. **Reading.** Before falling asleep, Alicia read from the top of page 37 to the top of page 62 of her book. How many pages did she read?

70. **Writing.** During a weekend retreat, James wrote from the bottom of page 29 to the bottom of page 37 of his memoirs. How many pages did he write?

71. Through exercise, Rod went from 8 lb above his “ideal” body weight to 9 lb below it. How many pounds did Rod lose?

72. Laura has a charge of $476.89 on her credit card, but she then returns a sweater that cost $128.95. How much does she now owe on her credit card?
73. Surface Temperatures on Mars. Surface temperatures on Mars vary from $-128{}^\circ C$ during polar night to $27{}^\circ C$ at the equator during midday at the closest point in orbit to the sun. Find the difference between the highest value and the lowest value in this temperature range.
Source: Mars Institute

74. Carla is completing the production work on a track that is to appear on her band’s upcoming CD. In doing so, she resets the digital recorder to 0, advances the recording 16 sec, and then reverses the recording 25 sec. What reading will the recorder then display?

75. While recording a 60-minute television show, the reading on Kate’s VCR changes from $-21$ min to 29 min. How many minutes have been recorded? Has she recorded the entire show?

76. As a result of coaching, Cedric’s average golf score improved from 3 over par to 2 under. By how many strokes did his score change?

77. Temperature Changes. One day the temperature in Lawrence, Kansas, is $32{}^\circ C$ at 6:00 A.M. It rises 15° by noon, but falls 50° by midnight when a cold front moves in. What is the final temperature?

78. Midway through a movie, Lisa resets the counter on her DVD player to 0. She then reverses the disc 8 min, and then advances the movie 11 min. What does the counter now read?


80. Tallest Mountain. The tallest mountain in the world, when measured from base to peak, is Mauna Kea (White Mountain) in Hawaii. From its base 19,684 ft below sea level in the Hawaiian Trough, it rises 33,480 ft. What is the elevation of the peak?
Source: The Guinness Book of Records
81. **Offshore Oil.** In 1998, the elevation of the world’s deepwater drilling record was $-7718$ ft. In 2005, the deepwater drilling record was $2293$ ft deeper. What was the elevation of the deepwater drilling record in 2005?  

**Source:** [www.deepwater.com/FactsandFirsts.cfm](http://www.deepwater.com/FactsandFirsts.cfm)

82. **Oceanography.** The deepest point in the Pacific Ocean is the Marianas Trench, with a depth of $11,033$ m. The deepest point in the Atlantic Ocean is the Puerto Rico Trench, with a depth of $8648$ m. What is the difference in the elevation of the two trenches?

83. **Toll Roads.** The Murrays began a trip with $13$ in their E-Z Pass account (see Example 11). They accumulated $20$ in tolls and had to pay $80$ in fines and administrative fees. By how much were the Murrays in debt as a result of their travel on toll roads?

84. **Toll Roads.** Suppose the Murrays (see Exercise 83) incurred $25$ in tolls and $85$ in fines and administrative fees. By how much would the Murrays be in debt?

85. **Dw** Write a subtraction problem for a classmate to solve. Design the problem so that the solution is “Clara ends up $15$ in debt.”

86. **Dw** If a negative number is subtracted from a positive number, will the result always be positive? Why or why not?

---

**SKILL MAINTENANCE**

Evaluate.

87. $4^3$  
88. $68 \cdot 72$  

89. $1^2$  
90. $143 \cdot 29$

91. How many 12-oz cans of soda can be filled with 96 oz of soda?  

92. A case of soda contains 24 bottles. If each bottle contains 12 oz, how many ounces of soda are in the case?

Simplify.

93. $5 + 4^2 + 2 \cdot 7$  
94. $45 \div (2^2 + 11)$

95. $(9 + 7)(9 - 7)$  
96. $(13 - 2)(13 + 2)$
SYNTHESIS

97. \textbf{Dw} Explain why the commutative law was used in Example 10.

98. \textbf{Dw} Is subtraction of integers associative? Why or why not?

Subtract.

99. $123,907 - 433,789$

100. $23,011 - (-60,432)$

For Exercises 101–106, tell whether each statement is true or false for all integers $a$ and $b$. If false, show why.

101. $a - 0 = 0 - a$

102. $0 - a = a$

103. If $a \neq b$, then $a - b \neq 0$.

104. If $a = -b$, then $a + b = 0$.

105. If $a + b = 0$, then $a$ and $b$ are opposites.

106. If $a - b = 0$, then $a = -b$.

107. If $a - 54$ is $-37$, find the value of $a$.

108. If $x - 48$ is $-15$, find the value of $x$.

109. Doreen is a stockbroker. She kept track of the weekly changes in the stock market over a period of 5 weeks. By how many points (pts) had the market risen or fallen over this time?

<table>
<thead>
<tr>
<th>WEEK 1</th>
<th>WEEK 2</th>
<th>WEEK 3</th>
<th>WEEK 4</th>
<th>WEEK 5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Down 13 pts</td>
<td>Down 16 pts</td>
<td>Up 36 pts</td>
<td>Down 11 pts</td>
<td>Up 19 pts</td>
</tr>
</tbody>
</table>

110. Blackjack Counting System. The casino game of blackjack makes use of many card-counting systems to give players an advantage if the count becomes negative. One such system is called High–Low, first developed by Harvey Dubner in 1963. Each card counts as $-1$, 0, or 1 as follows:

- 2, 3, 4, 5, 6 count as $+1$;
- 7, 8, 9 count as 0;
- 10, J, Q, K, A count as $-1$.

\textbf{Source:} Patterson, Jerry L. Casino Gambling. New York: Perigee, 1982

\textbf{a)} Find the total count on the sequence of cards

K, A, 2, 4, 5, 10, J, 8, Q, K, 5.

\textbf{b)} Does the player have a winning edge?
MULTIPLICATION OF INTEGERS

Objectives

2. Complete, as in the example.
3. Complete, as in the example.

Multiply.

1. \(4 \cdot 10 = 40\)
2. \(3 \cdot 10 = 30\)
3. \(2 \cdot 10 = 20\)
4. \(1 \cdot 10 = 10\)
5. \(0 \cdot 10 = 0\)
6. \(-1 \cdot 10 = -10\)
7. \(-2 \cdot 10 = -20\)
8. \(-3 \cdot 10 = -30\)

Multiply.

2. \(-3 \cdot 6\)

3. \(20 \cdot (-5)\)

4. \(9(-1)\)

5. Complete, as in the example.

Answers on page A-5

MULTIPLICATION OF A POSITIVE INTEGER AND A NEGATIVE INTEGER

To see how to multiply a positive integer and a negative integer, consider the following pattern.

This number decreases by 1 each time.  \[\begin{align*}
4 \cdot 5 &= 20 \\
3 \cdot 5 &= 15 \\
2 \cdot 5 &= 10 \\
1 \cdot 5 &= 5 \\
0 \cdot 5 &= 0 \\
-1 \cdot 5 &= -5 \\
-2 \cdot 5 &= -10 \\
-3 \cdot 5 &= -15 \\
\end{align*}\]

Do Exercise 1.

According to this pattern, it looks as though the product of a negative integer and a positive integer is negative. To confirm this, use repeated addition:

\[-1 \cdot 5 = 5 \cdot (-1) = -1 + (-1) + (-1) + (-1) + (-1) = -5\]
\[-2 \cdot 5 = 5 \cdot (-2) = -2 + (-2) + (-2) + (-2) + (-2) = -10\]
\[-3 \cdot 5 = 5 \cdot (-3) = -3 + (-3) + (-3) + (-3) + (-3) = -15\]

MULTIPLYING A POSITIVE AND A NEGATIVE INTEGER

To multiply a positive integer and a negative integer, multiply their absolute values and make the answer negative.

EXAMPLES Multiply.

1. \(8(-5) = -40\)
2. \(50(-1) = -50\)
3. \(-7 \cdot 6 = -42\)

Do Exercises 2–4.

MULTIPLICATION OF TWO NEGATIVE INTEGERS

How do we multiply two negative integers? Again we look for a pattern.

This number decreases by 1 each time.  \[\begin{align*}
4 \cdot (-5) &= -20 \\
3 \cdot (-5) &= -15 \\
2 \cdot (-5) &= -10 \\
1 \cdot (-5) &= -5 \\
0 \cdot (-5) &= 0 \\
-1 \cdot (-5) &= 5 \\
-2 \cdot (-5) &= 10 \\
-3 \cdot (-5) &= 15 \\
\end{align*}\]

Do Exercise 5.
According to the pattern, the product of two negative integers is positive. This leads to the second part of the rule for multiplying integers.

**MULTIPLYING TWO NEGATIVE INTEGERS**
To multiply two negative integers, multiply their absolute values. The answer is positive.

### EXAMPLES

**Multiply.**

4. \((-2)(-4) = 8\)
5. \((-10)(-7) = 70\)
6. \((-9)(-1) = 9\)

Do Exercises 6–8.

The following is another way to state the rules for multiplication.

To multiply two integers:

a) Multiply the absolute values.

b) If the signs are the same, the answer is positive.

c) If the signs are different, the answer is negative.

### MULTIPLICATION BY ZERO

No matter how many times 0 is added to itself, the answer is 0. This leads to the following result.

For any integer \(a\),

\[ a \cdot 0 = 0. \]

(The product of 0 and any integer is 0.)

### EXAMPLES

**Multiply.**

7. \(-19 \cdot 0 = 0\)
8. \(0(-7) = 0\)

Do Exercises 9 and 10.

### Multiplication of More Than Two Integers

Because of the commutative and the associative laws, to multiply three or more integers, we can group as we please.

### EXAMPLES

**Multiply.**

9. a) \(-8 \cdot 2(-3) = -16(-3) = 48\)  
   Multiplying the first two numbers

b) \(-8 \cdot 2(-3) = 24 \cdot 2 = 48\)  
   Multiplying the negatives

The result is the same as above.
10. \(7(-1)(-4)(-2) = (-7)8\)  

Multiplying the first two numbers and the last two numbers  

\[= -56\]

11. a) \(-5 \cdot (-2) \cdot (-3) \cdot (-6) = 10 \cdot 18\)  

Each pair of negatives gives a positive product.  

\[= 180\]

b) \(-5 \cdot (-2) \cdot (-3) \cdot (-6) \cdot (-1) = 10 \cdot 18 \cdot (-1)\)  

Making use of Example 11(a)  

\[= -180\]

We can see the following pattern in the results of Examples 9–11.

12. \((4)(-5)(-2)(-3)(-1)\)

Simplify.

13. \((-1)(-1)(-2)(-3)(-1)(-1)\)

Simplify.

14. \((-2)^3\)

15. \((-9)^2\)

16. \((-1)9\)

17. \(2^5\)

Answers on page A-5

Do Exercises 11–13.

POWERS OF INTEGERS

A positive number raised to any power is positive. When a negative number is raised to a power, the sign of the result depends upon whether the exponent is even or odd.

**EXAMPLES**  
Simplify.

12. \((-7)^2 = (-7)(-7) = 49\)  

The result is positive.

13. \((-4)^3 = (-4)(-4)(-4)\)  

\[= 16(-4)\]

\[= -64\]

The result is negative.

14. \((-3)^4 = (-3)(-3)(-3)(-3)\)  

\[= 9 \cdot 9\]

\[= 81\]

The result is positive.

15. \((-2)^5 = (-2)(-2)(-2)(-2)(-2)\)  

\[= 4 \cdot 4 \cdot (-2)\]

\[= 16(-2)\]

\[= -32\]

The result is negative.

Perhaps you noted the following.

When a negative number is raised to an even exponent, the result is positive.  
When a negative number is raised to an odd exponent, the result is negative.

Do Exercises 14–17.

When an integer is multiplied by \(-1\), the result is the opposite of that integer.

For any integer \(a\),  

\[-1 \cdot a = -a\]
EXAMPLE 16  Simplify: $-7^2$.

Since $-7^2$ lacks parentheses, the base is 7, not $-7$. Thus we regard $-7^2$ as $-1 \cdot 7^2$.

$$-7^2 = -1 \cdot 7^2$$
$$= -1 \cdot 7 \cdot 7$$

The rules for order of operations tell us to square first.

$$= -1 \cdot 49$$
$$= -49.$$ 

Compare Examples 12 and 16 and note that $(-7)^2 \neq -7^2$. In fact, the expressions $(-7)^2$ and $-7^2$ are not even read the same way: $(-7)^2$ is read “negative seven squared,” whereas $-7^2$ is read “the opposite of seven squared.”

Do Exercises 18–20.

CALCULATOR CORNER

Exponential Notation  When using a calculator to calculate expressions like $(-39)^4$ or $-39^4$, it is important to use the correct sequence of keystrokes.

Calculators with $+/-$ key: On some calculators, a $+/-$ key must be pressed after a number is entered to make the number negative. For these calculators, appropriate keystrokes for $(-39)^4$ are

$$3 \ 9 \ +/- \ ^4 \ x \ = \.$$ 

To calculate $-39^4$, we must first raise 39 to the power 4. Then the sign of the result must be changed. This can be done with the keystrokes

$$3 \ 9 \ ^4 \ x \ +/- \ = \.$$ 

or by multiplying $39^4$ by $-1$:

$$3 \ 9 \ ^4 \ x \ 1 \ +/- \ = \.$$ 

Calculators with $(-)$ key: On some calculators, the $(-)$ key is pressed before a number to indicate that the number is negative. This is similar to the way the expression is written on paper. For these calculators, $(-39)^4$ is found by pressing

$$1 \ (-) \ 3 \ 9 \ ^4 \ 1 \ \ \ \ \ \ = \.$$ 

and $-39^4$ is found by pressing

$$(-) \ 3 \ 9 \ ^4 \ = \.$$ 

You can either experiment or consult a user’s manual if you are unsure of the proper keystrokes for your calculator.

Exercises: Use a calculator to determine each of the following.

1. $(-23)^6$  
2. $(-17)^5$  
3. $(-104)^3$  
4. $(-4)^{10}$  
5. $-9^6$  
6. $-7^6$  
7. $-6^5$  
8. $-3^9$

Answers on page A-5

### Multiply.

<table>
<thead>
<tr>
<th>Exercise</th>
<th>Expression</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>$-2 \cdot 8$</td>
</tr>
<tr>
<td>2.</td>
<td>$-7 \cdot 3$</td>
</tr>
<tr>
<td>3.</td>
<td>$-9 \cdot 2$</td>
</tr>
<tr>
<td>4.</td>
<td>$-7 \cdot 7$</td>
</tr>
<tr>
<td>5.</td>
<td>$8 \cdot (-6)$</td>
</tr>
<tr>
<td>6.</td>
<td>$8 \cdot (-3)$</td>
</tr>
<tr>
<td>7.</td>
<td>$-10 \cdot 3$</td>
</tr>
<tr>
<td>8.</td>
<td>$-9 \cdot 8$</td>
</tr>
<tr>
<td>9.</td>
<td>$-3(-5)$</td>
</tr>
<tr>
<td>10.</td>
<td>$-8 \cdot (-2)$</td>
</tr>
<tr>
<td>11.</td>
<td>$-9 \cdot (-2)$</td>
</tr>
<tr>
<td>12.</td>
<td>$(8)(-9)$</td>
</tr>
<tr>
<td>13.</td>
<td>$(-6)(-7)$</td>
</tr>
<tr>
<td>14.</td>
<td>$-8 \cdot (-3)$</td>
</tr>
<tr>
<td>15.</td>
<td>$-10(-2)$</td>
</tr>
<tr>
<td>16.</td>
<td>$-9(-8)$</td>
</tr>
<tr>
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<td>$12(-10)$</td>
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<td>$15(-8)$</td>
</tr>
<tr>
<td>19.</td>
<td>$-6(-50)$</td>
</tr>
<tr>
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<td>$-25(-8)$</td>
</tr>
<tr>
<td>21.</td>
<td>$(-72)(-1)$</td>
</tr>
<tr>
<td>22.</td>
<td>$41(-3)$</td>
</tr>
<tr>
<td>23.</td>
<td>$(-20)17$</td>
</tr>
<tr>
<td>24.</td>
<td>$(-1)43$</td>
</tr>
<tr>
<td>25.</td>
<td>$-47 \cdot 0$</td>
</tr>
<tr>
<td>26.</td>
<td>$-17 \cdot 0$</td>
</tr>
<tr>
<td>27.</td>
<td>$0(-14)$</td>
</tr>
<tr>
<td>28.</td>
<td>$0(-38)$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Exercise</th>
<th>Expression</th>
</tr>
</thead>
<tbody>
<tr>
<td>29.</td>
<td>$3 \cdot (-8) \cdot (-1)$</td>
</tr>
<tr>
<td>30.</td>
<td>$(-7) \cdot (-4) \cdot (-1)$</td>
</tr>
<tr>
<td>31.</td>
<td>$7(-4)(-3)5$</td>
</tr>
<tr>
<td>32.</td>
<td>$9(-2)(-6)7$</td>
</tr>
<tr>
<td>33.</td>
<td>$-2(-5)(-7)$</td>
</tr>
<tr>
<td>34.</td>
<td>$(-2)(-5)(-3)(-5)$</td>
</tr>
<tr>
<td>35.</td>
<td>$(-5)(-2)(-3)(-1)$</td>
</tr>
<tr>
<td>36.</td>
<td>$-6(-5)(-9)$</td>
</tr>
<tr>
<td>37.</td>
<td>$(-15)(-29)0 \cdot 8$</td>
</tr>
<tr>
<td>38.</td>
<td>$19(-7)(-8)0 \cdot 6$</td>
</tr>
<tr>
<td>39.</td>
<td>$(-7)(-1)(7)(-6)$</td>
</tr>
<tr>
<td>40.</td>
<td>$(-5)6(-4)5$</td>
</tr>
</tbody>
</table>
Simplify.

41. \((-6)^2\)  
42. \((-8)^2\)  
43. \((-5)^3\)  
44. \((-2)^4\)

45. \((-10)^4\)  
46. \((-1)^5\)  
47. \(-2^4\)  
48. \((-2)^6\)

49. \((-3)^5\)  
50. \(-10^4\)  
51. \((-1)^{12}\)  
52. \((-1)^{13}\)

53. \(-3^6\)  
54. \(-2^6\)  
55. \(-4^3\)  
56. \(-2^5\)

Write each of the following expressions in words.

57. \(-8^4\)  
58. \((-6)^8\)  
59. \((-9)^{10}\)  
60. \(-5^4\)

61. \(\text{D}_W\) Explain in your own words why \((-9)^{10}\) is positive.

62. \(\text{D}_W\) Explain in your own words why \(-9^{10}\) is negative.

**SKILL MAINTENANCE**

63. Round 532,451 to the nearest hundred. \([1.4a]\)

64. Write standard notation for sixty million. \([1.1c]\)

65. Divide: \(2880 \div 36.\) \([1.6c]\)

66. Multiply: \(75 \times 34.\) \([1.5a]\)

67. Simplify: \(10 - 2^3 + 6 \div 2.\) \([1.9c]\)

68. Simplify: \(2 \cdot 5^2 - 3 \cdot 2^3 \div (3 + 2^3).\) \([1.9c]\)

69. A rectangular rug measures 5 ft by 8 ft. What is the area of the rug? \([1.5c, 1.8a]\)

70. How many 12-egg cartons can be filled with 2880 eggs? \([1.8a]\)

71. A ferry can accommodate 12 cars and 53 cars are waiting to go up. How many trips will be required to transport all of them? \([1.8a]\)

72. An elevator can hold 16 people and 50 people are waiting. How many trips will be required to ferry all of them? \([1.8a]\)

**SYNTHESIS**

73. \(\text{D}_W\) Which number is larger, \((-3)^{79}\) or \((-5)^{79}\)? Why?

Simplify.

75. \((-3)^3(-1)^{379}\)  
76. \((-2)^3 \cdot (-1)^{29}\)  
77. \(-9^4 + (-9)^4\)

79. \(|(-2)^{5} + 3^2| - (3 - 7)^2\)  
80. \(|-12(-3)^2 - 5^3 - 6^2 - (-5)^2|\)

81. \(-47^2\)  
82. \(-53^2\)  
83. \((-19)^4\)  
84. \((-23)^4\)

85. \((73 - 86)^3\)  
86. \((-49 + 34)^3\)  
87. \(-935(238 - 243)^3\)  
88. \((-17)^4(129 - 133)^5\)

89. Jo wrote seven checks for $13 each. If she had a balance of $68 in her account, what was her balance after writing the checks?

91. What must be true of \(m\) and \(n\) if \([(-5)^m]^{n}\) is to be (a) negative? (b) positive?

90. After diving 95 m below the surface, a diver rises at a rate of 7 meters per minute for 9 min. What is the diver’s new elevation?

92. What must be true of \(m\) and \(n\) if \(-mn\) is to be (a) positive? (b) zero? (c) negative?
2.5 DIVISION OF INTEGERS AND ORDER OF OPERATIONS

We now consider division of integers. Because of the way in which division is defined, its rules are similar to those for multiplication.

**a Division of Integers**

**THE QUOTIENT**

The quotient \( \frac{a}{b} \) (or \( a \div b \), or \( a/b \)) is the number, if there is one, that when multiplied by \( b \) gives \( a \).

Let's use the definition to divide integers.

**EXAMPLES** Divide, if possible. Check each answer.

1. \( 14 \div (-7) = -2 \)
   **Think:** What number multiplied by \(-7\) gives 14?
   The number is \(-2\). **Check:** \((-2) \cdot (-7) = 14\).

2. \( \frac{-32}{-4} = 8 \)
   **Think:** What number multiplied by \(-4\) gives \(-32\)?
   The number is 8. **Check:** \(8 \cdot 4 = -32\).

3. \( -21 \div 7 = -3 \)
   **Think:** What number multiplied by 7 gives \(-21\)?
   The number is \(-3\). **Check:** \((-3) \cdot 7 = -21\).

4. \( \frac{0}{-5} = 0 \)
   **Think:** What number multiplied by \(-5\) gives 0?
   The number is 0. **Check:** \(0 \cdot (-5) = 0\).

5. \( \frac{-5}{0} \) is not defined.
   **Think:** What number multiplied by 0 gives \(-5\)?
   There is no such number because the product of 0 and any number is 0.

The rules for determining the sign of a quotient are the same as for determining the sign of a product. We state them together.

To multiply or divide two integers:

- **a)** Multiply or divide the absolute values.
- **b)** If the signs are the same, the answer is positive.
- **c)** If the signs are different, the answer is negative.

**Do Exercises 1–6.**

Recall that, in general, \( a \div b \) and \( b \div a \) are different numbers. In Example 4, we divided into 0. In Example 5, we attempted to divide by 0. Since any number times 0 gives 0, not \(-5\), we say that \(-5 \div 0\) is **not defined** or is **undefined**. Also, since **any** number times 0 gives 0, \(0 \div 0\) is also not defined.

Answers on page A-5
EXCLUDING DIVISION BY 0

Division by 0 is not defined:

\[ a \div 0, \text{ or } \frac{a}{0}, \text{ is undefined for all real numbers } a. \]

DIVIDING 0 BY OTHER NUMBERS

Note that 0 \div 8 = 0 because 0 = 0 \cdot 8.

DIVIDENDS OF 0

Zero divided by any nonzero real number is 0:

\[ \frac{0}{a} = 0, \quad a \neq 0. \]

**EXAMPLES** Divide.

6. \(0 \div (-6) = 0\)

7. \(\frac{0}{12} = 0\)

8. \(-\frac{3}{0}\) is undefined.

Do Exercises 7–9.

**Rules for Order of Operations**

When more than one operation appears in a calculation or problem, we apply the same rules that were used in Section 1.9. We repeat them here for review, now including absolute-value symbols.

**RULES FOR ORDER OF OPERATIONS**

1. Do all calculations within parentheses, brackets, braces, absolute-value symbols, numerators, or denominators.
2. Evaluate all exponential expressions.
3. Do all multiplications and divisions in order from left to right.
4. Do all additions and subtractions in order from left to right.

**EXAMPLES** Simplify.

9. \(17 - 10 \div 2 \cdot 4\)

With no grouping symbols or exponents, we begin with the third rule.

\[
\begin{align*}
17 - 10 \div 2 \cdot 4 &= 17 - 5 \cdot 4 \\
&= 17 - 20 \\
&= -3
\end{align*}
\]

Carrying out all multiplications and divisions in order from left to right

10. \(|(-2)^3 \div 4| - 5(-2)|\)

We first simplify within the absolute-value symbols.

\[
\begin{align*}
|(-2)^3 \div 4| - 5(-2) &= |-8 \div 4| - 5(-2) \\
&= |-2| - 5(-2) \\
&= 2 - 5(-2) \\
&= 2 - (-10) \\
&= 12
\end{align*}
\]

Dividing

Multiplying

Subtracting
EXAMPLE 11  Simplify: $2^4 + 51 \cdot 4 - (37 + 23 \cdot 2)$.

$$
2^4 + 51 \cdot 4 - (37 + 23 \cdot 2)
= 2^4 + 51 \cdot 4 - (37 + 46) \quad \text{Carrying out all operations inside parentheses first, multiplying 23 by 2, following the rules for order of operations within the parentheses}
= 2^4 + 51 \cdot 4 - 83 \quad \text{Completing the addition inside parentheses}
= 16 + 51 \cdot 4 - 83 \quad \text{Evaluating exponential expressions}
= 16 + 204 - 83 \quad \text{Doing all multiplications}
= 220 - 83 \quad \text{Doing all additions and subtractions in order from left to right}
= 137
$$

Always regard a fraction bar as a grouping symbol. It separates any calculations in the numerator from those in the denominator.

EXAMPLE 12  Simplify: $\frac{5 - (-3)^2}{-2}$.

$$
\frac{5 - (-3)^2}{-2} = \frac{5 - 9}{-2} \quad \text{Calculating within the numerator:}
\quad (-3)^2 = (-3)(-3) = 9 \quad \text{and} \quad 5 - 9 = -4
\quad \frac{-4}{-2}
= \frac{2}{1} \quad \text{Dividing}
$$


CALCULATOR CORNER

Grouping Symbols  Most calculators now provide grouping symbols. Such keys may appear as $[($ and $])$ or $[\ldots]$ and $\ldots])$. Grouping symbols can be useful when we are simplifying expressions written in fraction form. For example, to simplify

$$\frac{38 + 142}{2 - 47},$$

we press $[3][8][+][1][4][2][÷][2][−][4][7][=]$.

Failure to include grouping symbols in the above keystrokes would mean that we are simplifying a different expression:

$$38 + \frac{142}{2} - 47.$$

Exercises: Use a calculator with grouping symbols to simplify each of the following.

1. $\frac{38 - 178}{5 + 30}$

2. $\frac{311 - 17^2}{2 - 13}$

3. $\frac{785 - 285 - 5^4}{17 + 3 \cdot 51}$

Answers on page A-5
Divide, if possible, and check each answer by multiplying. If an answer is undefined, state so.

1. \( 28 \div (-4) \)  
2. \( \frac{35}{-7} \)  
3. \( \frac{28}{2} \)  
4. \( 26 \div (-13) \)

5. \( \frac{18}{2} \)  
6. \( -22 \div (-2) \)  
7. \( \frac{-48}{-12} \)  
8. \( -63 \div (-9) \)

9. \( \frac{-72}{8} \)  
10. \( \frac{-50}{25} \)  
11. \( -100 \div (-50) \)  
12. \( -\frac{400}{8} \)

13. \( -344 \div 8 \)  
14. \( -\frac{128}{8} \)  
15. \( \frac{200}{-25} \)  
16. \( -651 \div (-31) \)

17. \( -\frac{56}{0} \)  
18. \( \frac{0}{-5} \)  
19. \( \frac{88}{-11} \)  
20. \( -\frac{145}{-5} \)

21. \( -\frac{276}{12} \)  
22. \( -\frac{217}{7} \)  
23. \( \frac{0}{-2} \)  
24. \( -\frac{13}{0} \)

25. \( -\frac{19}{1} \)  
26. \( -\frac{17}{1} \)  
27. \( -41 \div 1 \)  
28. \( 23 \div (-1) \)

Simplify, if possible. If an answer is undefined, state so.

29. \( 5 - 2 \cdot 3 - 6 \)  
30. \( 5 - (2 \cdot 3 - 7) \)  
31. \( 9 - 2(3 - 8) \)  
32. \( (8 - 2)(3 - 9) \)

33. \( 16 \cdot (-24) + 50 \)  
34. \( 10 \cdot 20 - 15 \cdot 24 \)  
35. \( 40 - 3^2 - 2^3 \)  
36. \( 2^4 + 2^2 - 10 \)
37. $4 \cdot (6 + 8)/(4 + 3)$  
38. $4^3 + 10 \cdot 20 + 8^2 - 23$  
39. $4 \cdot 5 - 2 \cdot 6 + 4$  
40. $5^3 + 4 \cdot 9 - (8 + 9 \cdot 3)$  

41. $\frac{9^2 - 1}{1 - 3^2}$  
42. $\frac{100 - 6^2}{(-5)^2 - 3^2}$  
43. $8(-7) + 6(-5)$  
44. $10(-5) + 1(-1)$  

45. $20 \div 5(-3) + 3$  
46. $14 \div 2(-6) + 7$  
47. $18 - 0(3^2 - 5^2 \cdot 7 - 4)$  
48. $9 \cdot 0 \div 5 \cdot 4$  

49. $4 \cdot 5^2 \div 10$  
50. $(2 - 5)^2 \div (-9)$  
51. $(3 - 8)^2 \div (-1)$  
52. $3 - 3^2$  

53. $17 - 10^3$  
54. $30 + (-5)^3$  
55. $2 + 10^2 \div 5 \cdot 2^2$  
56. $5 + 6^3 \div 3 \cdot 2^2$  

57. $12 - 20^3$  
58. $20 + 4^3 \div (-8)$  
59. $2 \times 10^3 - 5000$  
60. $-7(3^4) + 18$  

61. $6[9 - (3 - 4)]$  
62. $8[(6 - 13) - 11]$  
63. $-1000 \div (100) \div 10$  
64. $256 + (-32) \div (-4)$  

65. $8 - |7 - 9| \cdot 3$  
66. $|8 - 7 - 9| \cdot 2 + 1$  
67. $9 - |7 - 3^2|  
68. $9 - |5 - 7|^3$  

69. $\frac{6^3 - 7 \cdot 3^4 - 2^5 \cdot 9}{(1 - 2)^3 + 7^3}$  
70. $\frac{4 + 2 \cdot 4^2 - 3 \cdot 2}{(7 - 4)^3 - 2 \cdot 5 - 4}$  
71. $\frac{2 \cdot 3^2 \div (3^2 - (2 + 1))}{5^2 - 6^2 - 2^4(-3)}$  
72. $\frac{5 \cdot 6^2 \div (2^2 \cdot 5) - 7^2}{3^2 - 4^2 - (-2)^3 - 2}$  

73. $\frac{(-5)^3 + 17}{10(2 - 6) - 2(5 + 2)}$  
74. $\frac{(3 - 5)^2 - (7 - 13)}{(2 - 5)^3 + 2 \cdot 4}$  
75. $\frac{2 \cdot 4^3 - 4 \cdot 32}{19^3 - 17^4}$  
76. $\frac{-16 \cdot 28 \div 2^2}{5 \cdot 25 - 5^3}$  

77. $D_W$ Explain in your own words why $17 \div 0$ is undefined.  
78. $D_W$ Without performing any calculations, Stefan reports that $(19^2 - 17^2)/(16^2 - 18^2)$ is negative. How do you think he reached this conclusion?
79. Fabrikant Fine Diamonds ran a 4-in. by 7-in. advertisement in The New York Times. Find the area of the ad. \[1.5c\], \[1.8a\]

80. A classroom contains 7 rows of chairs with 6 chairs in each row. How many chairs are there in the classroom? \[1.8a\]

81. Cindi’s Ford Focus gets 32 mpg (miles per gallon). How many gallons will it take to travel 384 mi? \[1.8a\]

82. Craig’s Chevy Blazer gets 14 mpg. How many gallons will it take to travel 378 mi? \[1.8a\]

83. A 7-oz bag of tortilla chips contains 1050 calories. How many calories are in a 1-oz serving? \[1.8a\]

84. A 7-oz bag of tortilla chips contains 8 g (grams) of fat per ounce. How many grams of fat are in a carton containing 12 bags of chips? \[1.8a\]

85. There are 18 sticks in a large pack of Trident gum. If 4 people share the pack equally, how many whole pieces will each person receive? How many extra pieces will remain? \[1.8a\]

86. A bag of Ricola throat lozenges contains 24 cough drops. If 5 people share the bag equally, how many lozenges will each person receive? How many extra lozenges will remain? \[1.8a\]

87. \(D_W\) Ty claims that \(8 - 3^2 + 1\) is \(-2\). What mistake do you think he is making?

88. \(D_W\) Bryn contends that \(13 - 10/2 - 5\) is \(-1\). What mistake do you think she is making?

Simplify, if possible.

89. \[
\frac{9 - 3^2}{2 \cdot 4^2 - 5^2 \cdot 9 + 8^2 \cdot 7}
\]

90. \[
\frac{7^3 \cdot 9 - 6^2 \cdot 8 + 4^3 \cdot 6}{5^2 - 25}
\]

91. \[
\frac{(25 - 4^2)^3}{17^2 - 16^2} \cdot \frac{((-6)^2 - 6^2)}{7^2 - 8^2} \cdot (98 - 7^2 \cdot 2)
\]

92. \[
\frac{(7 - 8)^{37}}{7^2 - 8^2} \cdot (98 - 7^2 \cdot 2)
\]

93. \[
\frac{19 - 17^2}{13^2 - 34}
\]

94. \[
\frac{195 + (-15)^3}{195 - 7 \cdot 5^2}
\]

95. \[
28^2 - 36^2/4^2 + 17^2
\]

96. \[
9^3 - 36^3/12^2 + 9^2
\]

97. Write down the keystrokes needed to calculate \[
\frac{15^2 - 5^3}{3^2 + 4^2}.
\]

98. Write down the keystrokes needed to calculate \[
\frac{16^2 - 24 \cdot 23}{3 \cdot 4 + 5^2}.
\]

99. Evaluate the expression for which the keystrokes are as follows: \[
4 \quad 1 \quad 6 \quad + \quad 2 \quad + \quad 6 \quad = \quad 159.
\]

100. Evaluate the expression for which the keystrokes are \[
4 \quad 1 \quad 6 \quad + \quad 2 \quad + \quad 6 \quad = \quad 159.
\]

Determine the sign of each expression if \(m\) is negative and \(n\) is positive.

101. \[
\frac{-n}{m}
\]

102. \[
\frac{-n}{-m}
\]

103. \[
-\left(\frac{n}{m}\right)
\]

104. \[
-\left(\frac{n}{-m}\right)
\]

105. \[
-\left(\frac{-n}{-m}\right)
\]
In this section, we will learn to write equivalent expressions by making use of the distributive law, both of which are very important concepts.

**Algebraic Expressions**

In arithmetic, we work with expressions such as

\[ 37 + 86, \quad 7 \cdot 8, \quad 19 - 7, \quad \text{and} \quad \frac{3}{8}. \]

In algebra, we use both numbers and letters and work with algebraic expressions such as

\[ x + 86, \quad 7 \cdot t, \quad 19 - y, \quad \text{and} \quad \frac{a}{b}. \]

We have already worked with expressions like these.

When a letter can stand for various numbers, we call the letter a variable. A number or a letter that stands for just one number is called a constant.

An algebraic expression consists of variables, constants, numerals, and operation signs. When we replace a variable with a number, we say that we are substituting for the variable. This process is called evaluating the expression.

**EXAMPLE 1** Evaluate \( x + y \) for \( x = 37 \) and \( y = 29 \).

We substitute 37 for \( x \) and 29 for \( y \) and carry out the addition:

\[ x + y = 37 + 29 = 66. \]

The number 66 is called the value of the expression.

Algebraic expressions involving multiplication, like “8 times \( a \),” can be written as \( 8 \times a, \ 8 \cdot a, \ 8(a) \), or simply \( 8a \). Two letters written together without an operation symbol, such as \( ab \), also indicate multiplication.

**EXAMPLE 2** Evaluate \( 3y \) for \( y = -14 \).

\[ 3y = 3(-14) = -42 \quad \text{Parentheses are required here.} \]

Do Exercises 1–3.

Algebraic expressions involving division can also be written several ways.

For example, “8 divided by \( t \)” can be written as \( 8 \div t, \ 8/t, \) or \( \frac{8}{t} \).

**EXAMPLE 3** Evaluate \( \frac{a}{b} \) and \( -\frac{a}{-b} \) for \( a = 35 \) and \( b = 7 \).

We substitute 35 for \( a \) and 7 for \( b \):

\[ \frac{a}{b} = \frac{35}{7} = 5; \quad -\frac{a}{-b} = -\frac{35}{-7} = 5. \]
For each number, find two equivalent expressions with negative signs in different places.

4. \( \frac{-6}{x} \)

5. \( \frac{-m}{n} \)

6. \( \frac{r}{-4} \)

7. Evaluate \( \frac{a}{-b} \), \( \frac{-a}{b} \), and \( \frac{-a}{-b} \) for \( a = 28 \) and \( b = 4 \).

8. Find the Fahrenheit temperature that corresponds to 10 degrees Celsius (see Example 5).

9. Evaluate \( 3x^2 \) for \( x = 4 \) and \( x = -4 \).

10. Evaluate \( a^4 \) for \( a = 3 \) and \( a = -3 \).

11. Evaluate \( (-x)^2 \) and \( -x^2 \) for \( x = 3 \).

12. Evaluate \( (-x)^2 \) and \( -x^2 \) for \( x = 2 \).

13. Evaluate \( x^5 \) for \( x = 2 \) and \( x = -2 \).

**EXAMPLE 4** Evaluate \( \frac{-a}{b} \), \( \frac{-a}{b} \), and \( \frac{-a}{-b} \) for \( a = 15 \) and \( b = 3 \).

We substitute 15 for \( a \) and 3 for \( b \):

\[
\frac{-a}{b} = \frac{-15}{3} = -5; \quad \frac{-a}{b} = \frac{-15}{3} = -5; \quad \frac{-a}{-b} = \frac{15}{3} = -5.
\]

Examples 3 and 4 illustrate the following.

\[
-\frac{a}{b} \text{ and } \frac{a}{b} \text{ represent the same number.}
\]

\[
-\frac{a}{b} \text{ and } \frac{a}{b} \text{ all represent the same number.}
\]

Do Exercises 4–7.

**EXAMPLE 5** Evaluate \( \frac{9C}{5} + 32 \) for \( C = 20 \).

This expression can be used to find the Fahrenheit temperature that corresponds to 20 degrees Celsius:

\[
\frac{9C}{5} + 32 = \frac{9 \cdot 20}{5} + 32 = \frac{180}{5} + 32 = 36 + 32 = 68.
\]

Do Exercise 8.

**EXAMPLE 6** Evaluate \( 5x^2 \) for \( x = 3 \) and \( x = -3 \).

The rules for order of operations specify that the replacement for \( x \) be squared. That result is then multiplied by 5:

\[
5x^2 = 5(3)^2 = 5(9) = 45; \quad 5x^2 = 5(-3)^2 = 5(9) = 45.
\]

Example 6 illustrates that when opposites are raised to an even power, the results are the same.

Do Exercises 9 and 10.

**EXAMPLE 7** Evaluate \( (-x)^2 \) and \( -x^2 \) for \( x = 7 \).

When we evaluate \( (-x)^2 \) for \( x = 7 \), we have

\[
(-x)^2 = (-7)^2 = (-7)(-7) = 49. \quad \text{Substitute 7 for } x. \text{ Then evaluate the power.}
\]

To evaluate \( -x^2 \), we again substitute 7 for \( x \). We must recall that taking the opposite of a number is the same as multiplying that number by \(-1\).

\[
-x^2 = -1 \cdot x^2 \quad \text{The opposite of a number is the same as multiplying by } -1.
\]

\[
-7^2 = -1 \cdot 7^2 \quad \text{Substituting 7 for } x
\]

\[
= -1 \cdot 49 = -49. \quad \text{Using the rules for order of operations; calculating the power before multiplying}
\]

Example 7 shows that \( (-x)^2 \neq -x^2 \).

Do Exercises 11–13.
Equivalent Expressions and the Distributive Law

It is useful to know when two algebraic expressions will represent the same number. In many situations, this will help with problem solving.

**EXAMPLE 8** Evaluate \( x + x \) and \( 2x \) for \( x = 3 \) and \( x = -5 \) and compare the results.

We substitute 3 for \( x \) in \( x + x \) and again in \( 2x \):

\[
x + x = 3 + 3 = 6; \quad 2x = 2 \cdot 3 = 6.
\]

Next we repeat the procedure, substituting \(-5\) for \( x \):

\[
x + x = -5 + (-5) = -10; \quad 2x = 2(-5) = -10.
\]

The results can be shown in a table. It appears that \( x + x \) and \( 2x \) represent the same number.

<table>
<thead>
<tr>
<th>( x + x )</th>
<th>( 2x )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( x = 3 )</td>
<td>6</td>
</tr>
<tr>
<td>( x = -5 )</td>
<td>-10</td>
</tr>
</tbody>
</table>

Do Exercises 14 and 15.

Example 8 suggests that \( x + x \) and \( 2x \) represent the same number for any replacement of \( x \). When this is known to be the case, we can say that \( x + x \) and \( 2x \) are equivalent expressions.

**EQUIVALENT EXPRESSIONS**

Two expressions that have the same value for all allowable replacements are called equivalent.

In Examples 3 and 7 we saw that the expressions \(-\frac{a}{b}\) and \(\frac{a}{b}\) are equivalent but that the expressions \((-x)^2\) and \(-x^2\) are not equivalent.

An important concept, known as the distributive law, is useful for finding equivalent algebraic expressions. The distributive law involves two operations: multiplication and either addition or subtraction.

To review how the distributive law works, consider the following:

\[
\begin{align*}
4 \cdot 5 & = 20 \\
3 \cdot 7 & = 21 \\
2 \cdot 8 & = 16 \\
3 \cdot 15 & = 45
\end{align*}
\]

To carry out the multiplication, we actually added two products. That is,

\[
7 \cdot 45 = 7(40 + 5) = 7 \cdot 40 + 7 \cdot 5.
\]

The distributive law says that if we want to multiply a sum of several numbers by a number, we can either add within the grouping symbols and then multiply, or multiply each of the terms separately and then add.

**Answers on page A-5**
Use the distributive law to write an equivalent expression.

16. \(5(a + b)\)

**EXAMPLE 9** Evaluate \(a(b + c)\) and \(ab + ac\) for \(a = 3, b = 4,\) and \(c = 2.\)

We have

\[
a(b + c) = 3(4 + 2) = 3 \cdot 6 = 18 \quad \text{and} \quad \ab + ac = 3 \cdot 4 + 3 \cdot 2 = 12 + 6 = 18.
\]

It is impossible to overemphasize the importance of the parentheses in the statement of the distributive law. Were we to omit the parentheses, we would have \(ab + c.\) To see that \(a(b + c) \neq ab + c,\) note that \(3(4 + 2) = 18,\) but \(3 \cdot 4 + 2 = 14.\)

**EXAMPLE 10** Use the distributive law to write an expression equivalent to \(2(l + w).\)

\[
2(l + w) = 2 \cdot l + 2 \cdot w \quad \text{Note that the + sign between } l \text{ and } w \text{ now appears between } 2 \cdot l \text{ and } 2 \cdot w.
\]

\[
= 2l + 2w. \quad \text{Try to go directly to this step.}
\]

Do Exercises 16 and 17.

Since subtraction can be regarded as addition of the opposite, it follows that the distributive law holds in cases involving subtraction.

**EXAMPLE 11** Use the distributive law to write an expression equivalent to each of the following:

- **a)** \(7(a - b); \quad \text{b)} 9(x - 5); \quad \text{c)} (a - 7)b; \quad \text{d)} -4(x - 2y + 3z)\)

**a)** \(7(a - b) = 7 \cdot a - 7 \cdot b \quad \text{Try to go directly to this step.}\)

**b)** \(9(x - 5) = 9x - 9(5) \quad \text{Again, try to go directly to this step.}\)

**c)** \((a - 7)b = b(a - 7) \quad \text{Using a commutative law}\)

\[
= b \cdot a - b \cdot 7 \quad \text{Using the distributive law}\]

\[
= ab - 7b \quad \text{Using a commutative law to write } ba \text{ alphabetically and } b \cdot 7 \text{ with the constant first}\]

**d)** \(-4(x - 2y + 3z) = -4 \cdot x - (-4)(2y) + (-4)(3z) \quad \text{Using the distributive law}\)

\[
= -4x - (-4 \cdot 2)y + (-4 \cdot 3)z \quad \text{Using an associative law (twice)}\]

\[
= -4x - (-8y) + (-12z) \quad \text{Try to go directly to this step.}\]

Do Exercises 18–21.

---

*Answers on page A-5*
EXERCISE SET

Evaluate.

1. \(12n\), for \(n = 2\)  
   (The cost, in cents, of sending 2 text messages)

2. \(39n\), for \(n = 2\)  
   (The cost, in cents, of sending 2 letters)

3. \(\frac{x}{y}\), for \(x = 6\) and \(y = -3\)

4. \(\frac{m}{n}\), for \(m = 18\) and \(n = 2\)

5. \(\frac{2q}{p}\), for \(p = 6\) and \(q = 3\)

6. \(\frac{5y}{z}\), for \(y = 15\) and \(z = -25\)

7. \(\frac{72}{r}\), for \(r = 4\)
   (The approximate doubling time, in years, for an investment earning 4% interest per year)

8. \(\frac{72}{i}\), for \(i = 2\)
   (The approximate doubling time, in years, for an investment earning 2% interest per year)

9. \(3 + 5 \cdot x\), for \(x = 2\)

10. \(9 - 2 \cdot x\), for \(x = 3\)

11. \(2l + 2w\), for \(l = 3\) and \(w = 4\)
   (The perimeter, in feet, of a 3-ft by 4-ft rectangle)

12. \(3(a + b)\), for \(a = 2\) and \(b = 4\)

13. \(2(l + w)\), for \(l = 3\) and \(w = 4\)
   (The perimeter, in feet, of a 3-ft by 4-ft rectangle)

14. \(3a + 3b\), for \(a = 2\) and \(b = 4\)

15. \(7a - 7b\), for \(a = 5\) and \(b = 2\)

16. \(4x - 4y\), for \(x = 6\) and \(y = 1\)

17. \(7(a - b)\), for \(a = 5\) and \(b = 2\)

18. \(4(x - y)\), for \(x = 6\) and \(y = 1\)

19. \(16r^2\), for \(t = 5\)
   (The distance, in feet, that an object falls in 5 sec)

20. \(\frac{49t^2}{10}\), for \(t = 10\)
   (The distance, in meters, that an object falls in 10 sec)
21. \( a + (b - a)^2 \), for \( a = 6 \) and \( b = 4 \)

22. \((x + y)^2\), for \( x = 2 \) and \( y = 3 \)

23. \( 9a + 9b \), for \( a = 13 \) and \( b = -13 \)

24. \( 8x + 8y \), for \( x = 17 \) and \( y = -17 \)

25. \( \frac{n^2 - n}{2} \), for \( n = 9 \) (For determining the number of handshakes possible among 9 people)

26. \( \frac{5(F - 32)}{9} \), for \( F = 50 \) (For converting 50 degrees Fahrenheit to degrees Celsius)

27. \( m^3 - m^2 \), for \( m = 5 \)

28. \( a^6 - a \), for \( a = -2 \)

For each expression, write two equivalent expressions with negative signs in different places.

29. \( -\frac{5}{t} \)

30. \( \frac{7}{-x} \)

31. \( -\frac{n}{b} \)

32. \( -\frac{3}{r} \)

33. \( \frac{9}{-p} \)

34. \( -\frac{u}{5} \)

35. \( -\frac{14}{w} \)

36. \( -\frac{23}{m} \)

Evaluate \(-\frac{a}{b} - \frac{a}{b}\) and \(-\frac{a}{b}\) for the given values.

37. \( a = 45, b = 9 \)

38. \( a = 40, b = 5 \)

39. \( a = 81, b = 3 \)

40. \( a = 56, b = 7 \)
Evaluate.

41. \((-3x)^2\) and \(-3x^2\), for \(x = 2\)  

42. \((-2x)^2\) and \(-2x^2\), for \(x = 3\)

43. \(5x^2\), for \(x = 3\) and \(x = -3\)  

44. \(2x^2\), for \(x = 5\) and \(x = -5\)

45. \(x^3\), for \(x = 6\) and \(x = -6\)  

46. \(x^6\), for \(x = 2\) and \(x = -2\)

47. \(x^8\), for \(x = 1\) and \(x = -1\)  

48. \(x^3\), for \(x = 3\) and \(x = -3\)

49. \(a^2\), for \(a = 2\) and \(a = -2\)  

50. \(a^7\), for \(a = 1\) and \(a = -1\)

Use the distributive law to write an equivalent expression.

51. \(5(a + b)\)  

52. \(7(x + y)\)  

53. \(4(x + 1)\)

54. \(6(a + 1)\)  

55. \(2(b + 5)\)  

56. \(3(x - 6)\)

57. \(7(1 - t)\)  

58. \(4(1 - y)\)  

59. \(6(5x - 2)\)

60. \(9(6m + 7)\)  

61. \(8(x + 7 + 6y)\)  

62. \(4(5x + 8 + 3p)\)

63. \(-7(y - 2)\)  

64. \(-9(y - 7)\)  

65. \((x + 2)3\)

66. \((x + 4)2\)  

67. \(-4(x - 3y - 2z)\)  

68. \(8(2x - 5y - 8z)\)

69. \(8(a - 3b + c)\)  

70. \(-6(a + 2b - c)\)  

71. \(4(x - 3y - 7z)\)

72. \(5(9x - y + 8z)\)  

73. \((4a - 5b + c - 2d)5\)  

74. \((9a - 4b + 3c - d)7\)

75. \(D\)  Does \(-x^2\) always represent a negative number? Why or why not?

76. \(D\)  Is \(-x^2\) always negative? Why or why not?
77. Write a word name for 23,043,921. [1.1c]

78. Multiply: 17 \cdot 53. [1.5a]

79. Estimate by rounding to the nearest ten. Show your work. [1.4b]
\[
\begin{array}{c}
5283 \\
- 2475 \\
\end{array}
\]

80. Divide: 2982 \div 3. [1.6c]

81. On January 6, it snowed 9 in., and on January 7, it snowed 8 in. How much did it snow altogether? [1.8a]

82. On March 9, it snowed 12 in., but on March 10, the sun melted 7 in. How much snow remained? [1.8a]

83. For Brett's party, his wife ordered two cheese pizzas at $11 apiece and two pepperoni pizzas for $13 apiece. How much did she pay for the pizza? [1.8a]

84. For Tania's graduation party, her husband ordered three buckets of chicken wings at $12 apiece and 3 trays of nachos at $9 apiece. How much did he pay for the wings and nachos? [1.8a]

SYNTHESIS

85. D\textsubscript{W} Under what condition(s) will the expression ax\(^2\) be nonnegative? Explain.

86. D\textsubscript{W} Ted evaluates \(a + a^2\) for \(a = 5\) and gets 100 as the result. What mistake did he probably make?

87. A car’s catalytic converter works most efficiently after it is heated to about 370°C. To what Fahrenheit temperature does this correspond? (Hint: see Example 5.)

Evaluate.

88. \(x^2 - xy^2 + 2 \cdot y\), for \(x = 24\) and \(y = 6\)

89. \(a - b^3 + 17a\), for \(a = 19\) and \(b = -16\)

90. \(x^2 - 23y + y^3\), for \(x = 18\) and \(y = -21\)

91. \(r^3 + r^2t - rt^2\), for \(r = -9\) and \(t = 7\)

92. \(a^3b - a^2b^2 + ab^3\), for \(a = -8\) and \(b = -6\)

93. \(a^{1996} - a^{1997}\), for \(a = -1\)

94. \(x^{1492} - x^{1493}\), for \(x = -1\)

95. \((m^3 - mn)^m\), for \(m = 4\) and \(n = 6\)

96. \(5a^{3a-4}\), for \(a = 2\)

Replace the blanks with \(+, -, \times, \text{ or } \div\) to make each statement true.

97. \([-32 \square (88 \square 29)] = -1888\]

98. \([59 \square 17 \square 59 \square 8] = 1475\]

Classify each statement as true or false. If false, write an example showing why.

99. For any choice of \(x\), \(x^2 = (-x)^2\). \hspace{1cm} 100. For any choice of \(x\), \(x^3 = -x^3\).

101. For any choice of \(x\), \(x^6 + x^4 = (-x)^6 + (-x)^4\). \hspace{1cm} 102. For any choice of \(x\), \((-3x)^2 = 9x^2\).

103. D\textsubscript{W} If the Fahrenheit temperature is doubled, does it follow that the corresponding Celsius temperature is also doubled? (Hint: see Example 5.)
Like Terms and Perimeter

One common way in which equivalent expressions are formed is by combining like terms. In this section we learn how this is accomplished and apply the concept to geometry.

**Combining Like Terms**

A **term** is a number, a variable, a product of numbers and/or variables, or a quotient of numbers and/or variables. Terms are separated by addition signs. If there are subtraction signs, we can find an equivalent expression that uses addition signs.

**EXAMPLE 1**
What are the terms of $3xy - 4y + \frac{2}{z}$?

$3xy - 4y + \frac{2}{z} = 3xy + (-4y) + \frac{2}{z}$  
Separating parts with + signs

The terms are $3xy$, $-4y$, and $\frac{2}{z}$.

**Do Exercises 1 and 2.**

Terms in which the variable factors are exactly the same, such as $9x$ and $-4x$, are called **like**, or **similar**, terms. For example, $3y^2$ and $7y^2$ are like terms, whereas $5x$ and $6x^2$ are not. Constants, like 7 and 3, are also like terms.

**EXAMPLES**

2. $7x + 5x^2 + 2x + 8 + 5x^3 + 1$
   
   $7x$ and $2x$ are like terms; 8 and 1 are like terms.

3. $5ab + a^3 - a^2b - 2ab + 7a^3$
   
   $5ab$ and $-2ab$ are like terms; $a^3$ and $7a^3$ are like terms.

**Do Exercises 3 and 4.**

When an algebraic expression contains like terms, an equivalent expression can be formed by combining, or collecting, like terms. To combine like terms, we rely on the distributive law.

**EXAMPLE 4**
Combine like terms to form an equivalent expression.

a) $4x + 3x$  
   $\Rightarrow (4 + 3)x = 7x$  
   Using the distributive law (in “reverse”)

b) $6mn - 7mn$  
   $\Rightarrow (6 - 7)mn$  
   Try to do this mentally.

c) $7y - 2 - 3y + 5$  
   $\Rightarrow 7y + (-2) + (-3y) + 5$  
   Rewriting as addition

   $\Rightarrow 7y + (-3y) + (-2) + 5$  
   Using a commutative law

   $\Rightarrow 4y + 3$  
   Try to go directly to this step.

**Answers on page A-6**
Combine like terms to form an equivalent expression.

5. \(2a + 7a\)

6. \(5x^2 - 9 + 2x^2 + 3\)

7. \(4m - 2n^2 + 5 + n^2 + m - 9\)

Find the perimeter of each polygon.

8. \(\text{A polygon with five sides is called a pentagon.}\)

9. \(\text{A polygon with five sides is called a pentagon.}\)

10. Find the perimeter of a rectangle that is 2 cm by 4 cm.

Answers on page A-6
The perimeter of the rectangle in Example 6 is \(2 \cdot 3 \text{ cm} + 2 \cdot 4 \text{ cm},\) or equivalently \(2(3 \text{ cm} + 4 \text{ cm}).\) This can be generalized, as follows.

**PERIMETER OF A RECTANGLE**

The perimeter \(P\) of a rectangle of length \(l\) and width \(w\) is given by

\[ P = 2l + 2w, \quad \text{or} \quad P = 2 \cdot (l + w). \]

**EXAMPLE 7** A common door size is 3 ft by 7 ft. Find the perimeter of such a door.

\[
P = 2l + 2w \quad \text{We could also use } P = 2(l + w).
\]

\[
= 2 \cdot 7 \text{ ft} + 2 \cdot 3 \text{ ft} \quad \text{Try to do this mentally.}
\]

\[
= (2 \cdot 7) \text{ ft} + (2 \cdot 3) \text{ ft}
\]

\[
= 14 \text{ ft} + 6 \text{ ft}
\]

\[
= 20 \text{ ft}
\]

The perimeter of the door is 20 ft.

*Do Exercise 11.

A square is a rectangle in which all sides have the same length.

**EXAMPLE 8** Find the perimeter of a square with sides of length 9 mm.

\[
P = 9 \text{ mm} + 9 \text{ mm} + 9 \text{ mm} + 9 \text{ mm}
\]

\[
= (9 + 9 + 9 + 9) \text{ mm} \quad \text{Note that}
\]

\[
= 36 \text{ mm}
\]

*Do Exercise 12.

**PERIMETER OF A SQUARE**

The perimeter \(P\) of a square is four times \(s\), the length of a side:

\[ P = s + s + s + s = 4s. \]

**EXAMPLE 9** Find the perimeter of a square garden with sides of length 12 ft.

\[ P = 4s \]

\[ = 4 \cdot 12 \text{ ft} \]

\[ = 48 \text{ ft} \]

The perimeter of the garden is 48 ft.

*Do Exercise 13.

11. Find the perimeter of a 4-ft by 8-ft sheet of plywood.

12. Find the perimeter of a square with sides of length 10 km.

13. Find the perimeter of a square sandbox with sides of length 6 ft.

*Answers on page A-6*

**Study Tips**

**UNDERSTAND YOUR MISTAKES**

When you receive a graded quiz, test, or assignment back from your instructor, it is important to review and understand what your mistakes were. Too often students simply file away old papers without first making an effort to learn from their mistakes.
The goal of these matching questions is to practice step (2), Translate, of the five-step problem-solving process. Translate each word problem to an equation and select a correct translation from equations A–O.

A. \( 75 + x = 120 \)
B. \( 15 \div 750 = x \)
C. \( -10 - 15 = x \)
D. \( 8 \cdot 120 = x \)
E. \( 150 - 75 = x \)
F. \( 15 \cdot 750 = x \)
G. \( 75 - (-150) = x \)
H. \( 2 \cdot 150 + 2 \cdot 75 = x \)
I. \( -10 - (-15) = x \)
J. \( 8 \cdot x = 120 \)
K. \( 750 \div 15 = x \)
L. \( 15 - (-10) = x \)
M. \( 75 + 120 = x \)
N. \( 75 - 150 = x \)
O. \( 75 = 120 + x \)

Answers on page A-6
List the terms of each expression.

1. $2a + 5b - 7c$
2. $4x - 6y + 7z$
3. $9mn - 6n + 8$
4. $7rs + 4s - 5$
5. $3x^2y - 4y^2 - 2z^3$
6. $4a^3b + ab^2 - 9b^3$

Combine like terms to form an equivalent expression.

7. $5x + 9x$
8. $9a + 7a$
9. $10a - 13a$
10. $-16x + x$
11. $2x + 6z + 9x$
12. $3a - 5b + 7a$
13. $27a + 70 - 40a - 8$
14. $42x - 6 - 4x + 2$
15. $9 + 5t + 7y - t - y - 13$
16. $8 - 4a + 9b + 7a - 3b - 15$
17. $a + 3b + 5a - 2 + b$
18. $x + 7y + 5 - 2y + 3x$
19. $-8 + 11a - 5b + 6a - 7b + 7$
20. $8x - 5x + 6 + 3y - y - 4$
21. $8x^2 + 3y - x^2$
22. $8y^3 - 3z + 4y^3$
23. $11x^4 + 2y^3 - 4x^4 - y^3$
24. $13a^5 + 9b^4 - 2a^5 - 4b^4$
25. \(9a^2 - 4a + a - 3a^2\)  
26. \(3a^2 + 7a^3 - a^2 + 5 + a^3\)  
27. \(x^3 - 5x^2 + 2x^3 - 3x^2 + 4\)

28. \(9xy + 4y^2 - 2xy + 2y^2 - 1\)  
29. \(7a^3 + 4ab - 5 - 7ab + 8\)  
30. \(8a^2b - 3ab^2 - 4a^2b + 2ab\)

31. \(9x^3y + 4xy^3 - 6xy^3 + 3xy\)  
32. \(3x^4 - 2y^4 + 8x^4y^4 - 7x^4 + 8y^4\)  
33. \(3a^6 - 9b^4 + 2a^6b^4 - 7a^6 - 2b^4\)

34. \(9x^6 - 5y^5 + 3x^6y - 8x^6 + 4y^5\)

b Find the perimeter of each polygon.

35.  
36.  
37.  
38.  
39.  
40.
**Exercise Set 2.7**

**Tennis Court.** A tennis court contains many rectangles. Use the diagram of a regulation tennis court to calculate the perimeters in Exercises 41–44.

41. The perimeter of a singles court
42. The perimeter of a doubles court
43. The perimeter of the rectangle formed by the service lines and the singles sidelines
44. The perimeter of the rectangle formed by a service line, a baseline, and the singles sidelines

45. Find the perimeter of a rectangular 8-ft by 10-ft bedroom.
46. Find the perimeter of a rectangular 3-ft by 4-ft doghouse.

47. Find the perimeter of a checkerboard that is 14 in. on each side.
48. Find the perimeter of a square skylight that is 2 m on each side.

49. Find the perimeter of a square frame that is 65 cm on each side.
50. Find the perimeter of a square garden that is 12 yd on each side.

51. Find the perimeter of a 12-ft by 20-ft rectangular deck.
52. Find the perimeter of a 40-ft by 35-ft rectangular backyard.

53. Explain in your own words what it means for two algebraic expressions to be equivalent.
54. Can the formula for the perimeter of a rectangle be used to find the perimeter of a square? Why or why not?
55. A box of Shaw’s Corn Flakes contains 510 grams (g) of corn flakes. A serving of corn flakes weighs 30 g. How many servings are in one box? [1.8a]

56. Estimate the difference by rounding to the nearest ten. [1.4b]

704
-486

57. Simplify. [1.9c]

58. Simplify. [1.9c]

59. Simplify. [1.9c]

Solve. [1.7b]

63. 25 = t + 9

64. 19 = x + 6

65. 45 = 3x

66. 50 = 2t

67. DW Does doubling the length of a square's side double the perimeter of the original square? Why or why not?

68. DW Why was it necessary to introduce the distributive law before discussing how to combine like terms?

69. Simplify. (Multiply and then combine like terms.)

70. Simplify. (Multiply and then combine like terms.)

71. Simplify. (Multiply and then combine like terms.)

72. Simplify. (Multiply and then combine like terms.)

73. Simplify. (Multiply and then combine like terms.)

74. Simplify. (Multiply and then combine like terms.)

75. In order to save energy, Andrea plans to run a bead of caulk sealant around 3 exterior doors and 13 windows. Each window measures 3 ft by 4 ft, each door measures 3 ft by 7 ft, and there is no need to caulk the bottom of each door. If each cartridge of caulk seals 56 ft and costs $5.95, how much will it cost Andrea to seal the windows and doors?

76. Eric is attaching lace trim to small tablecloths that are 5 ft by 5 ft, and to large tablecloths that are 7 ft by 7 ft. If the lace costs $1.95 per yard, how much will the trim cost for 6 small tablecloths and 6 large tablecloths?

77. A square wooden rack is used to store the 15 numbered balls as well as the cue ball in pool. If a pool ball has a diameter of 57 mm, find the inside perimeter of the storage rack.

78. A rectangular box is used to store six Christmas ornaments. Find the perimeter of such a box if each ornament has a diameter of 72 mm.
In Section 1.7, we learned to solve certain equations by writing a “related equation.” We now extend this approach to include negative integers, as well as equations that involve both addition and multiplication.

### The Addition Principle

In Section 1.7, we learned to solve an equation like \( x + 12 = 27 \) by writing the related subtraction, \( x = 27 - 12 \), or \( x = 15 \). Note that \( x = 15 \) is an equation, not a solution. Of course, the solution of the equation \( x = 15 \) is obviously 15. The solution of \( x + 12 = 27 \) is also 15. Because their solutions are identical, \( x = 15 \) and \( x + 12 = 27 \) are said to be equivalent equations.

**Equivalent Equations**

Equations with the same solutions are called equivalent equations.

It is important to be able to distinguish between equivalent expressions and equivalent equations.

- \( 6a \) and \( 4a + 2a \) are equivalent expressions because, for any replacement of \( a \), both expressions represent the same number.
- \( 3x = 15 \) and \( 4x = 20 \) are equivalent equations because any solution of one equation is also a solution of the other equation.

### Example 1

Classify each pair as either equivalent equations or equivalent expressions:

- **a)** \( 5x + 1 \); \( 2x - 4 + 3x + 5 \)
- **b)** \( x = -7 \); \( x + 2 = -5 \).

**a)** First note that these are expressions, not equations. To see if they are equivalent, we combine like terms in the second expression:

\[
2x - 4 + 3x + 5 = (2 + 3)x + (-4 + 5)
\]

Regrouping and using the distributive law

\[
= 5x + 1.
\]

We see that \( 2x - 4 + 3x + 5 \) and \( 5x + 1 \) are equivalent expressions.

**b)** First note that both \( x = -7 \) and \( x + 2 = -5 \) are equations. The solution of \( x = -7 \) is \(-7\). We substitute to see if \(-7\) is also the solution of \( x + 2 = -5 \):

\[
x + 2 = -5
\]

\[
-7 + 2 = -5 \quad \text{TRUE}
\]

Since \( x = -7 \) and \( x + 2 = -5 \) have the same solution, they are equivalent equations.

Do Exercises 1 and 2.

There are principles that enable us to begin with one equation and create an equivalent equation similar to \( x = 15 \), for which the solution is obvious. One such principle, the addition principle, is stated on the next page.

Suppose that \( a \) and \( b \) stand for the same number and some number \( c \) is added to \( a \). We get the same result if we add \( c \) to \( b \), because \( a \) and \( b \) are equal.

---

**Objectives**

- Use the addition principle to solve equations.
- Use the division principle to solve equations.
- Decide which principle should be used to solve an equation.
- Solve equations that require use of both the addition principle and the division principle.

Classify each pair as equivalent expressions or equivalent equations.

1. \( a - 5 = -3; \quad a = 2 \)
2. \( a - 9 + 6a; \quad 7a - 9 \)
THE ADDITION PRINCIPLE

For any numbers $a$, $b$, and $c$,

\[ a = b \] is equivalent to \[ a + c = b + c. \]

**EXAMPLE 2** Solve: $x - 7 = -2$.

We have

\[
\begin{align*}
    x - 7 &= -2 \\
    x - 7 + 7 &= -2 + 7 & \text{Using the addition principle:} \\
    x + 0 &= 5 & \text{adding 7 to both sides} \\
    x &= 5.
\end{align*}
\]

The solution appears to be 5. To check, we use the original equation.

Check:

\[
\begin{align*}
    x - 7 &= -2 \\
    5 - 7 &= -2 \\
    -2 &= -2.
\end{align*}
\]

TRUE

The solution is 5.

Do Exercises 3 and 4.

**EXAMPLE 3** Solve: $23 = t + 7$.

We have

\[
\begin{align*}
    23 &= t + 7 \\
    23 - 7 &= t + 7 - 7 & \text{Using the addition principle to add} -7 \\
    16 &= t + 0 & \text{or to subtract 7 on both sides} \\
    16 &= t
\end{align*}
\]

The solution is 16. The check is left to the student.

To visualize the addition principle, think of a jeweler’s balance. When both sides of the balance hold equal amounts of weight, the balance is level. If weight is added or removed, equally, on both sides, the balance remains level.

Do Exercises 5 and 6.

Study Tips

**USE A PENCIL**

It is no coincidence that the students who experience the greatest success in this course work in pencil. We all make mistakes and by using pencil and eraser we are more willing to admit to ourselves that something needs to be rewritten. Please work with a pencil and eraser if you aren’t doing so already.
The Division Principle

In Section 1.7, we solved \(8n = 96\) by dividing both sides by 8:

\[
\begin{align*}
8 \cdot n &= 96 \\
\frac{8 \cdot n}{8} &= \frac{96}{8} \\
n &= 12.
\end{align*}
\]

Dividing both sides by 8.

You can check that \(8 \cdot n = 96\) and \(n = 12\) are equivalent. We can divide both sides of an equation by any nonzero number in order to find an equivalent equation.

**The Division Principle**

For any numbers \(a\), \(b\), and \(c (c \neq 0)\),

\[
\frac{a}{c} = \frac{b}{c}.
\]

In Chapter 3, after we have discussed multiplication of fractions, we will use an equivalent form of this principle: the multiplication principle.

**Example 4** Solve: \(9x = 63\).

We have

\[
\begin{align*}
9x &= 63 \\
\frac{9x}{9} &= \frac{63}{9} \\
9x &= 7.
\end{align*}
\]

Using the division principle to divide both sides by 9.

Check: \(9x = 63\)

\[
\begin{align*}
9 \cdot 7 &= 63 \\
63 &| \text{ TRUE}
\end{align*}
\]

The solution is 7.

Do Exercises 7 and 8.

**Example 5** Solve: \(48 = -8n\).

It is important to distinguish between an opposite, as we have in \(-8n\), and subtraction, as we had in \(x - 5 = 19\) (margin exercise 3). To undo multiplication by \(-8\), we use the division principle:

\[
\begin{align*}
48 &= -8n \\
\frac{48}{-8} &= \frac{-8n}{-8} \\
-6 &= n.
\end{align*}
\]

Dividing both sides by \(-8\).

Check: \(48 = -8n\)

\[
\begin{align*}
48 &= -8(-6) \\
48 &| \text{ TRUE}
\end{align*}
\]

The solution is \(-6\).
Be sure that you understand why the addition principle is used in Example 2 and the division principle is used in Example 5.

Do Exercises 9 and 10 on the previous page.

Equations like \(-x = 7\) or \(-t = -3\) often give students difficulty. One way to handle problems of this sort is to multiply both sides of the equation by \(-1\).

**EXAMPLE 6** Solve: \(-x = 7\).

To solve an equation like \(-x = 7\), remember that when an expression is multiplied or divided by \(-1\), its sign is changed. Here we multiply on both sides by \(-1\) to change the sign of \(-x\):

\[
\begin{align*}
-x &= 7 \\
(-1)(-x) &= (-1) \cdot 7 \\
x &= -7. \\
\end{align*}
\]

Multiplying both sides by \(-1\)

Note that \((-1)(-x)\) is the same as \((-1)(-1)x\).

Check:

\[
\begin{align*}
-x &= 7 \\
7 &\mid \text{TRUE}
\end{align*}
\]

The solution is \(-7\).

Another way to solve Example 6 is to note that \(-x = -1 \cdot x\). Then we can divide both sides by \(-1\):

\[
\begin{align*}
-x &= 7 \\
-1 \cdot x &= 7 \\
\frac{-1 \cdot x}{-1} &= \frac{7}{-1} \\
x &= -7. \\
\end{align*}
\]

Do Exercises 11 and 12.

**C** Selecting the Correct Approach

It is important for you to be able to determine which principle should be used to solve a particular equation.

**EXAMPLES** Solve.

7. \(39 = -3 + t\)

Note that \(-3\) is added to \(t\). To undo addition of \(-3\), we subtract \(-3\) or simply add \(3\) on both sides:

\[
\begin{align*}
3 + 39 &= 3 + (-3) + t \\
42 &= 0 + t \\
42 &= t.
\end{align*}
\]

Using the addition principle

Check:

\[
\begin{align*}
39 &= -3 + t \\
39 &\mid -3 + 42 \\
&\mid 39 \quad \text{TRUE}
\end{align*}
\]

The solution is \(42\).
8. \( 39 = -3t \)
   To undo multiplication by \(-3\), we divide by \(-3\) on both sides:
   \[
   \frac{39}{-3} = \frac{-3t}{-3} \quad \text{Using the division principle}
   \]
   \[-13 = t. \]
   Check: \[
   \begin{array}{ccc}
   39 & = & -3 \times (-13) \\
   39 & ? & 39 \\
   \hline
   \end{array}
   \]
   The solution is \(-13\).

Do Exercises 13–15.

Using the Principles Together

Suppose we want to determine whether \(7\) is the solution of \(5x - 8 = 27\). To check, we replace \(x\) with \(7\) and simplify.

Check: \[
\begin{array}{ccc}
5x - 8 & = & 27 \\
5 \cdot 7 - 8 & ? & 27 \\
35 - 8 & ? & 27 \\
27 & ? & 27 \\
\hline
\end{array}
\]
This shows that \(7\) is the solution.

Do Exercises 16 and 17.

In the check above, note that the rules for order of operations require that we multiply before we subtract (or add).

The rules for order of operations dictate that unless grouping symbols indicate otherwise, multiplication and division are performed before any addition or subtraction. Thus, to evaluate \(5x - 8\),
we select a value: \(x\)
then multiply by 5: \(5x\)
and then subtract 8: \(5x - 8\).

In Example 9, which follows, these steps are reversed to solve for \(x\):

We will add 8: \(5x - 8 + 8\),
then divide by 5: \(\frac{5x}{5}\)
and isolate \(x\): \(x\).

In general, the last step performed when calculating is the first step to be reversed when finding a solution.

13. \(-2x = -52\)
14. \(-2 + x = -52\)
15. \(x \cdot 7 = -28\)
16. Determine whether \(-9\) is the solution of \(7x + 8 = -55\).
17. Determine whether \(-6\) is the solution of \(4x + 3 = -25\).

Answers on page A-6
EXAMPLE 9  Solve: $5x - 8 = 27$.

We first note that the term containing $x$ is $5x$. To isolate $5x$, we add 8 on both sides:

\[
\begin{align*}
5x - 8 &= 27 \\
5x - 8 + 8 &= 27 + 8 & \text{Using the addition principle} \\
5x + 0 &= 35 & \text{Try to do this step mentally.} \\
5x &= 35.
\end{align*}
\]

Next, we isolate $x$ by dividing by 5 on both sides:

\[
\begin{align*}
5x &= 35 \\
\frac{5x}{5} &= \frac{35}{5} & \text{Using the division principle} \\
x &= 7.
\end{align*}
\]

The check was performed on the previous page. The solution is 7.

Do Exercise 18.

EXAMPLE 10  Solve: $38 = -9x + 2$.

We first isolate $-9x$ by subtracting 2 on both sides:

\[
\begin{align*}
38 &= -9x + 2 \\
38 - 2 &= -9x + 2 - 2 & \text{Subtracting 2 (or adding -2) from both sides} \\
36 &= -9x + 0 & \text{Try to do this step mentally.} \\
36 &= -9x.
\end{align*}
\]

Now that we have isolated $-9x$ on one side of the equation, we can divide by $-9$ to isolate $x$:

\[
\begin{align*}
36 &= -9x \\
\frac{36}{-9} &= \frac{-9x}{-9} & \text{Dividing both sides by -9} \\
-4 &= x. & \text{Simplifying}
\end{align*}
\]

Check:

\[
\begin{align*}
38 &= -9x + 2 \\
\frac{38}{-9} &= \frac{-9 \cdot (-4) + 2}{36 + 2} & \text{TRUE}
\end{align*}
\]

The solution is $-4$.

Do Exercise 19.

Answers on page A-6
Classify each pair as either equivalent expressions or equivalent equations.

1. \(2x = 10; 5x = 25\)
2. \(4x + 1; 6 + 4x - 5\)
3. \(7a - 3; 4a - 3 + 3a\)

4. \(7t = 14; 4t = 8\)
5. \(4r + 3; 8 + 4r - 5\)
6. \(2r - 7; r - 7 + r\)

7. \(x - 9 = 8; x + 3 = 20\)
8. \(t + 4 = 19; t - 6 = 9\)
9. \(3(t + 2); 5 + 3t + 1\)

10. \(2x = -14; x - 2 = -9\)
11. \(x + 4 = -8; 2x = -24\)
12. \(4(x - 7); 3x - 28 + x\)

Solve.

13. \(x - 6 = -9\)
14. \(x - 5 = -7\)
15. \(x - 4 = -12\)
16. \(x - 7 = 5\)

17. \(a + 7 = 25\)
18. \(x + 9 = -3\)
19. \(x + 8 = -6\)
20. \(t + 5 = 13\)

21. \(24 = t - 8\)
22. \(-9 = x + 3\)
23. \(-12 = x + 5\)
24. \(17 = n - 6\)

25. \(-5 + a = 12\)
26. \(3 = 17 + x\)
27. \(-8 = -8 + t\)
28. \(-7 + t = -7\)

Solve.

29. \(6x = -24\)
30. \(-8t = 40\)
31. \(-3t = 42\)
32. \(3x = 24\)

33. \(-7n = -35\)
34. \(64 = -2t\)
35. \(0 = 6x\)
36. \(-5n = -65\)

37. \(55 = -5t\)
38. \(-x = 83\)
39. \(-x = 56\)
40. \(-2x = 0\)

41. \(n(-4) = -48\)
42. \(-x = -475\)
43. \(-x = -390\)
44. \(n(-6) = -42\)
Solve.

45. \( t - 6 = -2 \)  
46. \( 3t = -45 \)  
47. \( 6x = -54 \)  
48. \( x + 9 = -15 \)

49. \( 15 = -x \)  
50. \( -13 = x - 4 \)  
51. \( -21 = x + 5 \)  
52. \( -42 = -x \)

53. \( 35 = -7t \)  
54. \( 7 + t = -18 \)  
55. \( -17x = 68 \)  
56. \( -34 = x + 10 \)

57. \( 18 + t = -160 \)  
58. \( -48 = t(-12) \)  
59. \( -27 = x + 23 \)  
60. \( -135 = -9t \)

Solve.

61. \( 5x - 1 = 34 \)  
62. \( 7x - 3 = 25 \)  
63. \( 4t + 2 = 14 \)  
64. \( 3t + 5 = 26 \)

65. \( 6a + 1 = -17 \)  
66. \( 8a + 3 = -37 \)  
67. \( 2x - 9 = -23 \)  
68. \( 3x - 5 = -35 \)

69. \( -2x + 1 = 17 \)  
70. \( -4t + 3 = -17 \)  
71. \( -8t - 3 = -67 \)  
72. \( -7x - 4 = -46 \)

73. \( -x + 9 = -15 \)  
74. \( -x - 6 = 8 \)  
75. \( 7 = 2x - 5 \)  
76. \( 9 = 4x - 7 \)

77. \( 13 = 3 + 2x \)  
78. \( 33 = 5 - 4x \)  
79. \( 13 = 5 - x \)  
80. \( 12 = 7 - x \)

81. **D W** To solve \(-5x = 13\), Eva decides to add 5 to both sides of the equation. Is there anything wrong with her doing this? Why or why not?

82. **D W** Gary decides to solve \(x - 9 = -5\) by adding 5 to both sides of the equation. Is there anything wrong with his doing this? Why or why not?
**SKILL MAINTENANCE**

**VOCABULARY REINFORCEMENT**

In each of Exercises 83–90, fill in the blank with the correct term from the given list. Some of the choices may not be used and some may be used more than once.

83. A(n) ________ is a closed geometric figure.  [2.7b]
84. Terms are ________ if they have the same variable factor(s).  [2.7a]
85. Numbers we multiply together are called ________ .  [1.5a]
86. Equations are ________ if they have the same solutions.  [2.8a]
87. The result of an addition is a(n) ________.  [1.2a]
88. A(n) ________ is a letter that can stand for various numbers.  [2.6a]
89. The ________ of a number is its distance from zero on a number line.  [2.1c]
90. We ________ for a variable when we replace it with a number.  [2.6a]

**SYNTHESIS**

91. **D_W** Explain how equivalent expressions can be used to write equivalent equations.

92. **D_W** To solve $2x + 8 = 24$, Wilma divides both sides by 2. Can this first step lead to a solution? Why or why not?

Solve.

93. $2x - 7x = -40$
94. $9 + x - 5 = 23$
95. $17 - 3^2 = 4 + t - 5^2$

96. $(-9)^2 = 2^3t + (3 \cdot 6 + 1)t$
97. $(-7)^2 - 5 = t + 4^3$
98. $(-42)^3 = 14^2t$

99. $x - (19)^3 = -18^3$
100. $23^2 = x + 22^2$
101. $35^3 = -125t$

102. $248 = 24 - 32x$
103. $529 - 143x = -1902$

---

The review that follows is meant to prepare you for a chapter exam. It consists of three parts. The first part, Concept Reinforcement, is designed to increase understanding of the concepts through true/false exercises. The second part is a list of important properties and formulas. The third part is the Review Exercises. These provide practice exercises for the exam, together with references to section objectives so you can go back and review. Before beginning, stop and look back over the skills you have obtained. What skills in mathematics do you have now that you did not have before studying this chapter?

**CONCEPT REINFORCEMENT**

Determine whether the statement is true or false. Answers are given at the back of the book.

1. The absolute value of a number is always nonnegative.  
2. The opposite of the opposite of a number is the original number.  
3. The product of an even number of negative numbers is positive.  
4. The expression \(2(x + 3)\) is equivalent to the expression \(2 \cdot x + 3\).  
5. \(3 - x = 4x\) and \(5x = -3\) are equivalent equations.  
6. Collecting like terms is based on the distributive law.

**IMPORTANT PROPERTIES AND FORMULAS**

For any integers \(a\), \(b\), and \(c\):

- \(a + (-a) = 0\); \(a - b = a + (-b)\); \(a \cdot 0 = 0\); \(a(b + c) = ab + ac\)
- **Perimeter of a Rectangle**: \(P = 2l + 2w\), or \(P = 2(l + w)\)
- **Perimeter of a Square**: \(P = 4s\)
- **The Addition Principle**: \(a = b\) is equivalent to \(a + c = b + c\).
- **The Division Principle**: For \(c \neq 0\), \(a = b\) is equivalent to \(\frac{a}{c} = \frac{b}{c}\).

**Review Exercises**

1. Tell which integers correspond to this situation: [2.1a]
   Bonnie has $527 in her campus account and Roger is $53 in debt.

Use either < or > for □ to form a true statement. [2.1b]

2. □ 0 □ -5  
3. □ -7 □ 6  
4. □ -4 □ -19
Find the absolute value.  
5. $|−39|$  
6. $|23|$  
7. $|0|$  

8. Find $−x$ when $x = −72$.  
9. Find $−(−x)$ when $x = 59$.  

Compute and simplify.  
10. $−14 + 5$  
11. $−5 + (−6)$  
12. $14 + (−8)$  
13. $0 + (−24)$  
14. $17 − 29$  
15. $9 − (−14)$  
16. $−8 − (−7)$  
17. $−3 − (−10)$  
18. $−3 + 7 + (−8)$  
19. $8 − (−9) − 7 + 2$  
20. $−23 · (−4)$  
21. $7(−12)$  
22. $2(−4)(−5)(−1)$  
23. $15 ÷ (−5)$  
24. $\frac{−55}{11}$  
25. $\frac{0}{7}$  

26. $7 ÷ 1^2 · (−3) − 4$  
27. $(−3)[4 − 3^2] − 5$  

28. Evaluate $3a + b$ for $a = 4$ and $b = −5$.  

29. Evaluate $\frac{−x}{y}, \frac{x}{−y}$, and $−\frac{x}{y}$ for $x = 30$ and $y = 5$.  

30. $4(5x + 9)$  
31. $3(2a − 4b + 5)$  

32. $5a + 12a$  
33. $−7x + 13x$  
34. $9m + 14 − 12m − 8$  

35. Find the perimeter of a rectangular frame that is 8 in. by 10 in.  
36. Find the perimeter of a square pane of glass that is 25 cm on each side.
Solve. \[2.8a, b, c, d\]

37. \(x - 9 = -17\)

38. \(-4t = 36\)

39. \(13 = -x\)

40. \(56 = 6x - 10\)

41. \(-x + 3 = -12\)

42. \(18 = 4 - 2x\)

43. **D_W** Explain the difference between equivalent expressions and equivalent equations. \[2.8a\]

44. **D_W** Is a number’s absolute value ever less than the number itself? Why or why not? \[2.1c\]

45. **D_W** A classmate insists on reading \(-x\) as “negative \(x\).” When asked why, the response is “because \(-x\) is negative.” What mistake is this student making? \[2.1d\]

46. **D_W** Are \((a - b)^2\) and \((b - a)^2\) equivalent for all choices of \(a\) and \(b\)? Why or why not? Experiment with different replacements for \(a\) and \(b\). \[2.6a\]

Simplify. \[2.5b\]

47. \(87 ÷ 3 \cdot 29^3 - (-6)^6 + 1957\)

48. \(1969 + (-8)^5 - 17 \cdot 15^3\)

49. \(\frac{113 - 17^3}{15 + 8^3 - 507}\)

50. For what values of \(x\) will \(8 + x^3\) be negative? \[2.6a\]

51. For what values of \(x\) is \(|x| > x^2\)? \[2.1b, c\]
1. Tell which integers correspond to this situation: The Tee Shop sold 542 fewer muscle shirts than expected in January and 307 more than expected in February.

2. Use either < or > for to form a true statement.
   \[ -14 \quad \underline{\text{<}} \quad -21 \]

3. Find the absolute value: \(|-739|\).

4. Find \((-x)\) when \(x = -19\).

Compute and simplify.

5. \(6 + (-17)\)  
6. \(-9 + (-12)\)  
7. \(-8 + 17\)

8. \(0 - 12\)  
9. \(7 - 22\)  
10. \(-5 - 19\)

11. \(-8 - (-27)\)  
12. \(31 - (-3) - 5 + 9\)  
13. \((-4)^3\)

14. \(27(-10)\)  
15. \(-9 \cdot 0\)  
16. \(-72 \div (-9)\)

17. \(\frac{-56}{7}\)  
18. \(8 \div 2 \cdot 2 - 3^2\)  
19. \(29 - (3 - 5)^2\)
20. **Antarctica Highs and Lows.** The continent of Antarctica, which lies in the southern hemisphere, experiences winter in July. The average high temperature is $-67^\circ$F and the average low temperature is $-81^\circ$F. How much higher is the average high than the average low?  
*Source:* National Climatic Data Center

21. Jeannie rewound a tape in her video camera from the 8 minute mark to the $-15$ minute mark. How many minutes of tape were rewound?

22. Evaluate $\frac{a - b}{6}$ for $a = -8$ and $b = 10$.

23. Use the distributive law to write an equivalent expression.  
$7(2x + 3y - 1)$

24. Combine like terms.  
$9x - 14 - 5x - 3$

Solve.

25. $-7x = -35$

26. $a + 9 = -3$

**SYNTHESIS**

27. Monty plans to attach trim around the doorway and along the base of the walls in a 12-ft by 14-ft room. If the doorway is 3 ft by 7 ft, how many feet of trim are needed? (Only three sides of a doorway get trim.)

Simplify.

28. $9 - 5[x + 2(3 - 4x)] + 14$

29. $15x + 3(2x - 7) - 9(4 + 5x)$

30. $49 \cdot 14^3 + 7^4 + 1926^2 + 6^2$

31. $3487 - 16 \div 4 \cdot 4 \div 2^8 \cdot 14^4$
1. Write standard notation for the number written in words in the following sentence: In 2003 there were about one hundred eighty-one million, five hundred ninety-nine thousand, nine hundred telephone lines in use in the United States.

2. Write a word name for 5,380,001,437.

Add.

3. \[15,892 + 2,935 = \]

4. \[7989 + 79 = \]

Subtract.

5. \[8276 - 430 = \]

6. \[3006 - 578 = \]

Multiply.

7. \[621 \times 27 = \]

8. \[2505 \times 3300 = \]

9. \[43 \cdot (-8) = \]

10. \[-12(-6) = \]

Divide.

11. \[63 \div 6552 = \]

12. \[62 \div 301,400 = \]

13. \[0 \div (-67) = \]

14. \[60 \div (-12) = \]

15. Round 427,931 to the nearest thousand.

16. Round 5309 to the nearest hundred.

17. Estimate each sum or product by rounding to the nearest hundred. Show your work.

18. \[749,559 + 301,362 = \]

19. Use < or > for □ to form a true sentence:

-26 □ 19.

20. Find the absolute value: \(|-279|\).

Simplify.

21. \(35 - 25 ÷ 5 + 2 \times 3\)  
22. \(\{17 - [8 - (5 - 2 \times 2)]\} ÷ (3 + 12 ÷ 6)\)

23. \(10 ÷ 1(-5) - 6^2\)  
24. \(5^3\)

25. Evaluate \(\frac{x + y}{5}\) for \(x = 11\) and \(y = 4\).  
26. Evaluate \(7x^2\) for \(x = -2\).

Use the distributive law to write an equivalent expression.

27. \(-2(x + 5)\)  
28. \(6(3x - 2y + 4)\)

Simplify.

29. \(-12 + (-14)\)  
30. \((-3)(-10)\)  
31. \(23 - 38\)  
32. \(64 ÷ (-2)\)

33. \(-12 - (-25)\)  
34. \((-2)(-3)(-5)\)  
35. \(3 - (-8) + 2 - (-3)\)  
36. \(16 ÷ 2(-8) + 7\)

Solve.

37. \(x + 8 = 35\)  
38. \(-12r = 36\)  
39. \(6 - x = -9\)  
40. \(-39 = 4x - 7\)

Solve.

41. In the movie Little Big Man, Dustin Hoffman plays a character who ages from 17 to 121. This represents the greatest age range depicted by one actor in one film. How many years did Hoffman’s character age?  
Source: Guinness Book of World Records

42. The ten largest hotels in the United States are in Las Vegas. Of these, the four largest are the MGM Grand, the Luxor, the Excalibur, and the Circus Circus. These have 5034 rooms, 4408 rooms, 4008 rooms, and 3770 rooms, respectively. What is the total number of rooms in these four hotels?  
Source: http://govegas.about.com/cs/hotels/tp/largesthotels.htm

43. Amanda is offered a part-time job paying $4940 a year. How much is each weekly paycheck?  
44. Eastside Appliance sells a refrigerator for $600 and $30 tax with no delivery charge. Westside Appliance sells the same model for $560 and $28 tax plus a $25 delivery charge. Which is the better buy?

45. Write an equivalent expression by combining like terms: \(7x - 9 + 3x - 5\).

SYNTHESIS

46. A soft-drink distributor has 166 loose cans of cola. The distributor wishes to form as many 24-can cases as possible and then, with any remaining cans, as many six-packs as possible. How many cases will be filled? How many six-packs? How many loose cans will remain?

47. Simplify: \(a - [3a - (4a - (2a - 4a))].\)

48. Simplify: \(37 \cdot 64 ÷ 4^2 \cdot 2 - (7^3 - (-4)^5).\)

49. Find two solutions of \(5|x| - 2 = 13.\)